Cyclical Fluctuations, Financial Frictions, and Productivity Differences across Firms*

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February 5, 2026

Abstract

Why do firms with vastly different levels of productivity coexist within narrowly defined industries? We build a tractable model that highlights the key role of credit rationing in limiting the reallocation of credit across firms with heterogeneous levels of productivity. Investors cannot distinguish between more productive and less productive firms. They invest an aliquot share of their savings in all firms. The less productive firms end up becoming financial intermediaries: They lend their share of financing from investors to more productive firms. But since the idiosyncratic productivity shocks are private information, credit does not flow efficiently to the most productive firms. Some low-productivity firms default strategically on their loans. We show that our model can match key data properties, such as the dispersion of productivity across firms and its evolution in the face of cyclical fluctuations.

JEL Classification: E31, E44
Keywords: productivity dispersion, financial intermediation, business cycles

* The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

1 Introduction

Why do firms with vastly different levels of productivity coexist within narrowly defined industries? We build a model that highlights the key role of credit rationing in limiting the reallocation of credit across firms with heterogeneous levels of productivity. We assume that firms experience both idiosyncratic and aggregate productivity shocks. Investors cannot distinguish between more productive and less productive firms. They invest an aliquot share of their savings in all firms. The less productive firms end up becoming financial intermediaries and lend their share of financing from investors to more productive firms. But since the idiosyncratic productivity shocks are private information, the most productive firms cannot borrow as much as they would like.

In our framework, moral hazard and asymmetric information result in misallocation of capital and reduced productivity. Moral hazard limits the borrowing capacity of the most efficient firm, allowing less efficient firms to borrow from intermediaries. Asymmetric information arises from uncertainty about borrower quality, as productivity above some threshold level is private and unobservable. While borrowing firms could offer to pay a higher loan rate to attract financing, this strategy lacks credibility, as all firms would claim to have high productivity. Similarly, lenders do not raise the lending rate to the efficient market-clearing level to avoid attracting borrowers that would strategically default. Accordingly, credit rationing arises in equilibrium, as in Stiglitz and Weiss (1981).

The model encompasses two sources of total factor productivity (TFP). Aside from a standard exogenous source, changes in productivity dispersion also affect TFP endogenously. Under our baseline calibration, this channel leads to meaningful differences in the response of the economy to exogenous TFP shocks relative to a model without financial frictions and productivity dispersion. The efficient allocation of resources would funnel all credit to the most productive firm, in which case our model collapses to a workhorse real business cycle model.

Despite its relative simplicity, the model we develop is sufficiently flexible to match some key data properties. The model is calibrated to match the average dispersion of productivity across firms, the relative importance of bank and nonbank credit, the average default rate of bank loans, and the average spread between the corporate rate and inter-bank loan rate.[1] The model comes close to replicating some key untargeted moments, such as the evolution of productivity dispersion over the business cycle, and its reaction to shocks that tighten financial conditions. Furthermore, as in the data, loan default rates are counter-cyclical.

Our work has many antecedents. Firstly, it is related to papers that showcase models with costly state verification, such as Carlstrom and Fuerst (1997), Bernanke et al. (1999), Christiano et al. (2014). All these papers build theories of the allocation of credit to firms of different productivity levels, but these levels and their dispersion are static — they do not respond to shocks. Furthermore, default decisions do not reflect strategic motives. Default stems from the inability to repay debts after the resolution of uncertainty reveals lower-than-expected productivity. By contrast, in our model, default is strategic, varies in response to changes in economic conditions, and affects the dispersion of productivity, apart from implying losses for lenders.

Our work is also related to more recent papers that include endogenously evolving productivity dispersion. Khan and Thomas (2013) were the first to explore the endogenous TFP channel in a quantitative DSGE setting in which real frictions slow the reallocation of capital across firms and in which reallocation is essential in determining aggregate TFP. Our model captures some elements of the misallocation of resources at the center of Khan and Thomas (2013), but not the slow reallocation friction. In their model, there is no equilibrium default. By contrast, we consider a strategic default in a tractable model. The simplicity of our framework allows us to embed it in the many variants of an RBC model. We view our modeling contribution as providing a building block that others can easily reuse or extend.

We share a focus on simplicity with Buera and Moll (2015). They develop a model with firms with different productivity levels that face a collateral constraint when borrowing. Shocks to that constraint can open up productivity wedges. Similarly, Liu and Wang (2014) and Dong et al. (2021) develop a model with heterogeneous firms subject to collateral constraints when borrowing. Relative to the models in these papers, while still tractable, our setup includes additional economic margins, such as endogenous loan default and multiple

[1]Many papers have documented the ubiquity of productivity dispersion across production units. We rely on Cunningham et al. (2023), but see also Hsieh and Klenow (2009), or Kehrig (2011).

sources of credit.

Our paper contributes to a body of work exploring the interactions of heterogeneous firms, financial frictions, and default decisions. Gilchrist et al. (2014) study credit spreads under uncertainty shocks in a model with default. Arellano et al. (2019) examine the role of uncertainty shocks in a model with noncontingent debt and equilibrium default. Gomes and Schmid (2021) develop a model with endogenous default, where firms vary with respect to their leverage, and study the implication for credit spreads.

Finally, our paper is related to an extensive literature explaining productivity differences across firms. Beyond financial frictions, the range of alternative explanations for productivity differences includes slow learning of the average level of productivity in the face of noise shocks, as in Jovanovic (1982); as in Caballero and Hammour (1994), Caballero and Hammour (1996) Caballero and Hammour (1998); search costs and matching efficiency between workers and jobs, as in Barlevy (2002), or imperfect product substitutability as in Bernard et al. (2003), and Melitz (2003).

2 Data Motivation

We document that there are sizable and pervasive within-industry productivity differences in the U.S. manufacturing sector. Then we show that the dispersion of TFP across establishments in the same industry is negatively correlated with GDP growth. We also document that the dispersion in productivity levels is negatively associated with the default rate on bank loans—the novel aspect of our theoretical model—and the real interest rate that we use as a measure of financial conditions. The summary statistics that we construct are the same ones that we will use to assess our theoretical model.

2.1 Data Description

We draw dispersion in productivity across 86 four-digit NAICS manufacturing industries from new experimental productivity dispersion statistics, Dispersion Statistics on Productivity (DiSP), derived from Census Bureau microdata and the statistics from the Bureau of Labor Statistics (BLS) built from industry-level aggregates.[2] The most recent release of DiSP covers the years 1987–2021 on annual basis. Our analysis focuses on the second-moment measure of establishment-level total factor productivity.

Figure 1 shows the evolution of TFP dispersion over time. The solid line denotes the average within-industry TFP dispersion. This average is weighted by each industry's contribution to GDP.[3] The top panel considers the percentage difference in productivity between the establishment at the 90^{th} percentile of the TFP distribution and the 10^{th} percentile establishment in the same industry. To interpret the values on the vertical axis, one hundred percent means that the plant at the 90^{th} percentile of the productivity distribution makes twice as much output with the same measured inputs as the 10^{th} percentile plant. As shown, the average TFP gap has been sizable and relatively stable, starting at around 95 percent in 1987 and moving gradually up to a little over 120 percent in 2021.

The dot-dashed bands capture the dispersion of the same productivity gap across industries. In a point-wise fashion (year by year), we consider the 90-10 gap for each industry and highlight the gap for the industry at the 15^{th} percentile and at the 85^{th} percentile. The range spanned by these percentiles is fairly symmetric. Moreover, the within-industry productivty gap is still sizable, even at the lower range across industries. The takeaway is that the average TFP gap across all industries is not swayed by industry outliers. Large within-industry productivity differences are pervasive.

For robustness, the bottom panel repeats the same analysis for the interquartile range (IQR). We conclude that within-industry productivity differences are important quantitatively for the outer segments of the distribution as well as for segments closer to the center of the distribution.

Turning to cyclical variation, the shaded areas in Figure 1 show recessions as dated by the National Bureau of Economic Research (NBER). Focusing on the average within industry gaps, no clear pattern is readily apparent. Regardless of whether we focus on The TFP gap between the 90^{th} and 10^{th} percentile plant, or the gap in the IQR, TFP dispersion rises in som recessions and falls on others. To check for systematic patterns taking into account the

[2]This dataset is described in Cunningham et al. (2023).

[3]The industry value-added data used to construct the weighted average TFP dispersion shown in Figure 1 is from the Bureau of Economic Analysis

full dataset, not just average TFP gaps, we turn to some regressions next.

2.2 Regression Results

To document how within-industry productivity dispersion is related to key macroeconomic indicators we resort to panel regressions. Our modest goal is to construct a set of moments that we can use to judge the empirical relevance of our model. We do not have loftier goals of establishing causal relationships. Our simple framework is the following:

$$\gamma_{i,t} = c_i + \beta x_t + \varepsilon_{i,t}, \tag{1}$$

where $\gamma_{i,t}$ is the within-industry TFP gap for industry i, either between the 90^{th} and 10^{th} percentile plants or the IQR. Industry-specific fixed effects are captured by the term c_i. The term x_t denotes alternative cyclical indicators, and ε_{it} is an industry-specific error term. Our regressions include industry weights proportional to the value added shares of the 86 industries included.[4]

Table 2 shows our regression results for alternative choices of the dependent variable, x_t. The first set of results in the table is for the annual growth rate of real GDP, expressed as a percentage.[5] The relationship is negative, implying that within-industry TFP dispersion is countercyclical. This finding is consistent with Cunningham et al. (2023), Bloom et al. (2018), and Kehrig (2011), among others.

In our second specification of Table 2, the dependent variable is the delinquency rate of business loans at commercial banks.[6] The coefficient is negative, implying that elevated delinquency rates are associated with lower within-industry TFP dispersion.

Different explanations are consistent with this relationship. One possible explanation for this association is that, delinquency rates tend to rise before outright defaults. To the extent

[4]To construct industry weights, we aggregate the value added of multiple NAICS manufacturing industries that correspond to the same BLS code.

[5]We use an annual chain-type quantity index of real gross domestic product from the National Income and Products Account (NIPA) of the U.S. Bureau of Economic Analysis.

[6]The data on delinquency rates on business loans are from the data release Charge-Offs and Delinquency Rates on Business Loans and Leases at Commercial Banks of the Board of Governors of the Federal Reserve System. This release is based on data from the FFIEC Consolidated Report on Conditions and Income. Delinquent loans and leases are defined as those at least thirty days past due. we constructed annual averages from quarterly data.

that lower productivity firms are more likely to default, one can expect TFP dispersion to be reduced by the default of unproductive firms.[7] But causality could also flow in the other direction.

In our third specification. as shown in Table 2, the dependent variable is a short-term real interest rate, which we take as a simple indicator of financial conditions.[8] In this case, the estimate of the slope coefficient β is negative, implying that less costly financing is associated with lower allocative efficiency. Of course, we could use a broader index of financial conditions, but we prefer this simple indicator given our goal of producing analogous moments with synthetic data from our simple model.

Across specifications, the regression coefficients are significant at the 1 percent level based on clustered standard errors.

For robustness, Table 3 reports analogous regression results using the IQR of TFP dispersion. Across specifications, the slope coefficient is significant at the 5 percent level, and the signs are unchanged.

3 Model

Our analysis focuses on the key role of credit markets in reallocating capital among heterogeneous firms. We assume that firms experience two types of productivity shocks: transitory idiosyncratic shocks and persistent aggregate shocks. The idiosyncratic shocks generate a distribution of firms with different levels of productivity on the unit interval. As this type of productivity is private information, households will allocate an aliquot share of their savings to each firm. We will show that the more-productive firms borrow from less-productive firms to purchase additional capital.

Unlike in a frictionless credit market, moral hazard and asymmetric information result

[7] The BLS Dispersion Statistics on Productivity do not allow to separate whether differences in TFP across quantiles of the within-industry distribution change because the lower quantiles rise or because the upper quantiles fall.

[8] Nominal interest rate data are for the 3-month Treasury bill on a discount basis from the secondary market as reported in the Federal Reserve Board's H.15 Release. The expected inflation is calculated from iterating one-period forward the inflation data for which we use the quarterly log change in the chain-type price index for personal consumption expenditures excluding food and energy from NIPA Table 2.3.4 of the U.S. Bureau of Economic Analysis. We then convert the calculated quarterly rate to an annual measure.

in misallocation of capital and reduced productivity in our framework. Moral hazard limits the borrowing capacity of the most efficient firm. Asymmetric information arises from uncertainty about borrower quality, as productivity above some threshold level is private and unobservable. Whereas the most productive firms could offer to pay a higher rate to borrow more funds, this strategy lacks credibility. Words are cheap, and all eligible firms could claim high productivity. Thus in our model, credit is rationed, as in Stiglitz and Weiss (1981).

Our interpretation of inter-firm lending reflects the broader role of credit markets in allocating credit. As Bernanke and Gertler (1990) note, this lending can be viewed as intermediated by competitive financial institutions that neither use resources nor earn profits in equilibrium. Variation in the cutoff points we just discussed will also lead to endogenous changes in the size of the intermediation sector.

Aside from the endogenous financial intermediation, the other main innovation of the model consists of tracking the dispersion of productivity across goods-producing firms in a tractable way. Relative to a standard model, the complications will consist of characterizing three additional equilibrium objects. The first two objects are a couple of cutoff points within the distribution of firm productivity. A lower cutoff point will separate firms with such low productivity that lending to more productive firms will be more profitable than producing. Just on the other side of this cutoff will be more-productive firms that borrow in the inter-firm market. An upper cutoff point will demarcate firms that borrow and default from those that do not default. The third additional object is the price at which funds are lent across firms.

As these cutoff points and the cost of credit fluctuate, productivity differences across firms will also fluctuate, as will default rates, together with the overall efficiency of the economy. We will show that despite the simplicity of our model, key characteristics of these fluctuations align well with U.S. data.

Firms are at the center of our model, but we start with households for ease of exposition. Before proceeding, we need to spell out some notation: Lowercase letters will denote variables of individual firms or households; we shall reserve uppercase letters for aggregate variables.

3.1 Households

There is an infinitely-lived representative household that has preferences over consumption and labor, respectively C_t and H_t. The preferences are in line with the specification in Greenwood et al. (1988). The household solves the following problem:

$$\max_{\left\{A_{t+\tau},C_{t+\tau},H_{t+\tau},B^H_{t+\tau}\right\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{1}{1-\sigma} \left(C_{t+\tau} - \vartheta \frac{H_{t+\tau}^{1+\nu}}{1+\nu} \right)^{1-\sigma}, \tag{2}$$

subject to

$$C_{t+\tau} + A_{t+\tau} + B^H_{t+\tau} = R^A_{t+\tau} A_{t+\tau-1} + W_{t+\tau} H_{t+\tau} + R^B_{t+\tau-1} B^H_{t+\tau-1} + \Pi_t + T_t + \Xi_t. \tag{3}$$

In period $t+\tau$, the household consumes $C_{t+\tau}$ units of good and chooses the amount of assets $A_{t+\tau}$ and a government bond $B^H_{t+\tau}$. The household enters period $t+\tau$ with assets $A_{t+\tau-1}$ which carry a state-contingent return $R^A_{t+\tau}$ from ownership of the banking sector and receives a non-state contingent return $R^B_{t+\tau-1}$ from its holdings of the government bond $B^H_{t+\tau-1}$. It also supplies $H_{t+\tau}$ units of labor at the wage rate $W_{t+\tau}$. The term Π_t captures profits from ownership of goods-producing and capital-producing firms (the latter could be non-zero in equilibrium), and the term T_t represents a lump-sum transfer from the government, which runs a balanced budget, period by period. Finally, the term Ξ_t refers to transfers that the household receives because firms that take the outside option, which is described below, are subject to a haircut on the returns from the outside option.

Below are the first-order conditions for assets

$$-\lambda_{ct} + \beta E_t \left\{ \lambda_{ct+1} R^A_{t+1} \right\} = 0, \tag{4}$$

consumption

$$\left(C_t - \vartheta \frac{H_t^{1+\nu}}{1+\nu} \right)^{-\sigma} = \beta E_t \left\{ \lambda_{ct+1} \left(R^B_t + FS_t \right) \right\}, \tag{5}$$

labor

$$-\vartheta \left(C_t - \vartheta \frac{H_t^{1+\nu}}{1+\nu} \right)^{-\sigma} H_t^{\nu} + \beta E_t \left\{ \lambda_{ct+1} \left(R^B_t + FS_t \right) \right\} W_t = 0, \tag{6}$$

government bonds

$$-\lambda_{ct}+\beta E_t\left\{\lambda_{ct+1}R_t^B\right\}=0, \tag{7}$$

where λ_{ct} is the Lagrange multiplier attached to the budget constraint in equation (3) and FS_t is a financial shock, inspired by the risk premium shock in Smets and Wouters (2007). The shock is specified as an exogenous term appended to the next period return in the consumption Euler equation. Note that we also insert FS_t in equation (6) to avoid distorting the intratemporal condition on labor supply. To wit, combining equations (5) and (7) and inserting the result into equation (6) yields the standar intratemporal labor supply condition, $W_t=\vartheta H_t^{\nu}$. This shock evolves according to

$$FS_t=\rho_f FS_{t-1}+\varepsilon_t^f. \tag{8}$$

NOtice that when the innovation term ε_t^f is negative, the households' required return on bonds increases, and so do the financing costs of firms.[9]

3.2 Production and Financial Intermediation

Firms in the goods sector are at the heart of our model. The firms in this sector follow a two-period overlapping structure from period t to $t+1$. This device is familiar from decentralizations of the RBC model that rely on equity contracts to allocate household savings to firms. Table 1 provides a roadmap to the sequence of choices made and actions taken by firms within each period. We highlight with a **boldface** font the choices or actions that are specific to our model.

Firms operate in a perfectly competitive market, producing a homogeneous good. Although identical ex ante, firms face idiosyncratic productivity shocks, $\omega\in[0,1]$, which they manage by borrowing from or lending to each other. We will show that some of the firms in this sector will endogenously become financial intermediaries.

In a slight abuse of notation, the term ω will do double duty by denoting the idiosyncratic level of productivity and by serving as an index for other firm-specific variables. Since

[9] Fisher (2015) shows that a similar shock can have a structural interpretation as a shock to the demand for safe and liquid assets.

households have no visibility into the idiosyncratic productivity of firms, each firm receives an aliquot share of the households' savings, a_t. So, notice that a_t does not depend on ω in equilibrium.

Table 1: Summary of firms' decisions and actions within each period

	Period t	Period $t+1$
1	Raise equity a_t	Produce, **outside option matures**
2	**Productivity level** $\omega \in [0,1]$ **is drawn**	Repay loans to other firms
3	**Lend or borrow in inter-firm market**	Pay households
4	**Some borrowing firms default and take outside option**	
5	Purchase physical capital	

Starting from the production function, the output of an individual firm in period $t+1$, $y_{t+1}(\omega)$, is governed by

$$y_{t+1}(\omega) = \omega Z_{t+1} k_t(\omega)^{\alpha} h_{t+1}(\omega)^{1-\alpha}. \tag{9}$$

The terms $k_t(\omega)$ and $h_{t+1}(\omega)$ denote the levels of capital and labor inputs used by the firm. The idiosyncratic productivity ω follows the cumulative distribution function $\mu(\omega)$ on the interval $[0,1]$, satisfying $\mu(0)=0$, $\mu(1)=1$, and $\mu'(\omega)>0$. The aggregate technology shock Z_{t+1} evolves according to

$$\log Z_{t+1} = \rho_z \log Z_t + \varepsilon^z_{t+1}, \tag{10}$$

where ε_{t+1} follows a Normal distribution with zero mean and standard deviation σ_z.

Firms make plans in period t to produce in period $t+1$. After their idiosyncratic productivity ω is known, the intermediation market opens. Depending on their productivity, some firms borrow while others lend at the predetermined rate ρ_t. Firms that borrow decide whether to use an outside option by purchasing the government bond or produce.[10] If they choose to produce, they purchase physical capital.

Firms can walk away from loans and default. If a firm decides to default, it can retain a fraction $\Theta_t(\omega)$ of the funds borrowed from other firms $b_t(\omega)$. We assume that $\Theta_t(\omega)$ increases in ω. For simplicity, we make the average fraction of funds that can be diverted equal to θ,

[10]Notice that lending firms can always decide to take the outside option, which pins down ρ_t.

so

$$\theta = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \Theta_t(\omega) \frac{\mu'(\omega)}{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)} \mathrm{d}\omega. \tag{11}$$

Creditor firms are assumed not to be able to go after any funds set aside for the outside option. The diverted assets are then placed with the outside option and yield $\left(R_t^B - \xi\right)(a_t(\omega) + \Theta_t(\omega) b_t^i(\omega))$ at time $t+1$. The term ξ is a haircut on the returns from the government bond R_t^B for taking the outside option. The haircut is rebated in a lump-sum fashion to households.

A screening technology allows lenders to tell whether borrowers can be expected to make more than the outside option by producing. This technology prevents the firms with the lowest levels of firm-specific technology from borrowing and defaulting and may support a mass of lenders, those with $\omega < \overline{\omega}_t$. Firms whose private productivity ω is between $\overline{\omega}_t$ and $\overline{\overline{\omega}}_t$ will find it advantageous to divert all available funds towards the outside option. These firms will earn $\left(R_t^B - \xi\right)(a_t(\omega) + \Theta_t(\omega) b_t(\omega))$. Only firms that can make higher returns by producing will produce.

With constant returns to scale production, the economy would attain the first-best allocation if the most efficient firm, the one with $\omega = 1$, could borrow all financing from other firms, allowing it to be the only firm to produce. This allocation is hindered by the presence of moral hazard and asymmetric information. By limiting the borrowing capacity of the most efficient firm, moral hazard gives less efficient firms room to borrow from financial intermediaries. Asymmetric information refers to uncertainty about the quality of borrowers.[11] The productivity level above $\overline{\omega}_t$ is private information that is not observed by other firms. Producing firms may seek more funds by offering to borrow above the prevailing lending rate. However this offer lacks credibility as all firms above a certain threshold could claim high productivity. The result is credit rationing, as in Stiglitz and Weiss (1981). Consequently, all inter-firm financial contracts are identical and do not depend on ω in equilibrium.

In the next three sections, we will consider three distinct optimization problems spanning the possible combination of actions that firms can take. Firms can borrow from other firms

[11] A fully accurate screening technology would deliver the first-best allocation with only the most productive firm producing. The assumption of an imperfect screening technology allows guarantees that low productivity firms will not switch to borrowing and defaulting.

and produce, borrow and default, or lend. We will show that firms sort themselves in each group depending on their idiosyncratic productivity, ω. For now, take this result as a hypothesis. The proof of the hypothesis is set up in Section 4 with details pushed to Appendix A.

3.2.1 Firms Choosing to Produce ($\omega \geq \overline{\overline{\omega}}_t$)

Firms acquire the capital stock required for production in period $t+1$, by combining funds from households $a_t(\omega)$ with a loan $b_t(\omega)$ from the mass of lending firms; consequently,

$$b_t^{tot}(\omega) = a_t(\omega) + b_t(\omega), \tag{12}$$

where $b_t^{tot}(\omega)$ denotes the total borrowing of the firm. By contractual agreement, the funds borrowed can only be used to purchase capital; so,

$$b_t^{tot}(\omega) = Q_t k_t(\omega), \tag{13}$$

where Q_t is the price of capital in terms of consumption. Let $R_{t+1}(\omega)$ denote the rate of return on capital ownership and $\pi_{t+1}(\omega)$ describe the profit of the firm. The revenue of the firm includes the proceeds from the sale of output as well as from the sale of the undepreciated fraction of capital. The expenses encompass commitments associated with loan servicing as well as remuneration for labor services. Thus,

$$\pi_{t+1}(\omega) = z_{t+1}\omega k_t(\omega)^{\alpha} h_{t+1}(\omega)^{1-\alpha} + (1-\delta)Q_{t+1}k_t(\omega) - R_{t+1}(\omega)b_t^{tot}(\omega) - W_{t+1}h_{t+1}(\omega), \tag{14}$$

where δ is the depreciation rate.

At time t, the problem of this producer of goods is to choose $b_t^{tot}(\omega)$ and $k_t(\omega)$ to maximize expected profits in period $t+1$, knowing that the firm will be able to choose the optimal amount of labor in that period. This maximization problem can be expressed as:

$$\max_{k_t(\omega), b_t^{tot}(\omega)} E_t \left\{ \max_{h_{t+1}(\omega)} \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \pi_{t+1}(\omega) \right\}, \tag{15}$$

where $\beta \frac{\lambda_{ct+1}}{\lambda_{ct}}$ is the stochastic discount factor coming from the household's problem. This maximization is subject to (13).

Due to constant returns to scale technology, both labor-to-capital and output-to-labor ratios are equalized across firms with different productivity levels and do not depend on ω. Thus, they coincide with the corresponding ratios of the aggregate variables. The Appendix derives the first-order conditions and characterizes the problem. It shows that

$$W_t = (1-\alpha)\frac{Y_t}{H_t}, \tag{16}$$

$$R_{t+1}(\omega) = \frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\omega + \frac{(1-\delta)}{Q_t}Q_{t+1} \tag{17}$$

under every state of nature. Equation (16) describes the demand for labor of the individual firm. Equation (17) is the zero-profit condition that can be interpreted as follows: $\frac{1}{Q_t}$ is the capital gained from one unit of consumption. This capital $\frac{1}{Q_t}$ earns a rental rate $\alpha Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\omega$ equal to the marginal product of capital. After production, the undepreciated part of capital can be resold at the price Q_{t+1}, so capital gains equal $\frac{(1-\delta)}{Q_t}Q_{t+1}$.

Upon observing ω, the producing firm determines the demand for inter-firm loans by solving the following problem:

$$\max_{b_t(\omega)|\omega} E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\Big(R_{t+1}(\omega)\left(a_t(\omega)+b_t(\omega)\right) - R^A_{t+1}(\omega)a_t(\omega) - \rho_t b_t(\omega)\Big)\right] \tag{18}$$

It generates returns from owning capital by utilizing borrowed funds and distributes these returns to meet equity commitments of households and repay loans from other firms.

Note that firms that produce, given constant returns to scale technology, would choose not to borrow in the inter-firm market if the returns to production were lower than the cost of inter-firm funding. The inter-firm rate is such that $\rho_t \leq E_t R_{t+1}$. Thus, this maximization problem implies that firms that borrow with the intent of producing will be interested in borrowing as much as possible. Accordingly, supply conditions will have to determine how much these firms can borrow.

We use the zero-profit condition that holds under every state of nature to size the return paid to households by firms in this segment, i.e.

$$R^A_{t+1}(\omega) = \frac{R_{t+1}(\omega)\left(a_t(\omega) + b_t(\omega)\right) - \rho_t b_t(\omega)}{a_t(\omega)}. \tag{19}$$

3.2.2 Firms Choosing to Borrow and Default ($\overline{\omega}_t \leq \omega < \overline{\overline{\omega}}_t$)

The problem of the firm that diverts the borrowed funds by choosing the outside option can be described as follows:

$$\max_{l_t(\omega)|\omega} E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \Big(\left(R^B_t - \xi\right)\left(a_t(\omega) + \Theta_t(\omega) b_t(\omega)\right) - R^A_{t+1}(\omega) a_t(\omega) \Big) \right]. \tag{20}$$

The firm earns returns by investing borrowed funds in the government bond and uses these to pay the returns on equity to households. When taking the outside option, a haircut ξ is applied to the yield R^B_t on the government bond. The term $\Theta_t(\omega)$ reflects that only a fraction $\Theta_t(\omega) > 0$ of the funds borrowed in the inter-firm market can be retained by the borrower when diverting the funds. Notice that this firm will be glad to borrow as much as possible, just as long as government bonds minus a haircut ξ pay a positive return. Therefore, similar to the previous situation, the supply conditions dictate the borrowing capacity of the firms in this segment.

We use the zero-profit condition that holds under every state of nature to size the return paid to households by firms in this segment, i.e.

$$R^A_{t+1}(\omega) = \frac{\left(R^B_t - \xi\right)\left(a_t(\omega) + \Theta_t(\omega) b_t(\omega)\right)}{a_t(\omega)}. \tag{21}$$

3.2.3 Firms Choosing to Lend ($\omega < \overline{\omega}_t$)

The problem of the firm that lends in the inter-firm market collapses to

$$\max_{l_t(\omega)|\omega} E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(\rho_t \frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)} l_t(\omega) + \frac{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)} (1-\theta) l_t(\omega) - R^A_{t+1}(\omega) a_t(\omega) \right) \right], \tag{22}$$

subject to the constraint that $l_t(\omega) \leq a_t(\omega)$.

The term $\dfrac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}$ represents the share of loans to firms that choose to produce, yielding the return ρ_t. The term $\dfrac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}$ represents the share of loans to firms that choose to divert the borrowed funds, yielding the average fraction that can be recovered 1-θ.[12] The firm pays the household the return on assets $R^A_{t+1}(\omega)$.

Notice that the revenues of the firm are increasing in the amount of loans it supplies. Thus, the budget constraint will hold with equality:

$$l_t(\omega) = a_t(\omega). \tag{23}$$

The first-order condition with respect to $l_t(\omega)$ is

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}(1-\theta)\right)\right]=0. \tag{24}$$

Notice that this first-order condition is implied by the stronger zero-profit condition that holds under every state of nature and that we use to size the return paid to households by firms in this segment, i.e.

$$R^A_{t+1}(\omega)=\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}(1-\theta). \tag{25}$$

3.3 Capital-Producing Firms

In period t, competitive capital-producing firms buy capital from the goods-producing firms, repair depreciated capital and build new capital. They sell both the new and re-furbished capital next period.

Let I^g_t denote aggregate gross investment expenditures. We introduce quadratic adjustment costs measured in units of investment such that the supply of investment goods is given by:

$$I^n_t=\left[1-\frac{\phi}{2}\left(\frac{I^g_t}{I^g_{t-1}}-1\right)^2\right]I^g_t, \tag{26}$$

where ϕ is a parameter that governs investment adjustment costs for current production

[12]Proposition 2 establishes that $\overline{\overline{\omega}}_t > \overline{\omega}_t$, ensuring that the share of loans to firms that choose to produce and the share of loans to firms that choose to divert the borrowed funds are non-negative and not greater than 1.

relative to past production.

The aggregate capital stock evolves according to:

$$K_t = I_t^n + (1-\delta)K_{t-1}, \tag{27}$$

where K_t is the amount of capital allocated to the goods-producing firms.

The capital producing firms are owned by households, and solve the problem

$$\max_{I_{t+i}^g} E_t \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{ct+i}}{\lambda_{ct}} \left\{ Q_{t+i} \left[1 - \frac{\phi}{2} \left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1 \right)^2 \right] I_{t+i}^g - I_{t+i}^g \right\}. \tag{28}$$

The first-order condition implies

$$0 = E_t \left\{ \begin{array}{c} Q_t \left[-\phi \left(\frac{I_t^g}{I_{t-1}^g} - 1 \right) \frac{1}{I_{t-1}^g} \right] I_t^g + Q_t \left[1 - \frac{\phi}{2} \left(\frac{I_t^g}{I_{t-1}^g} - 1 \right)^2 \right] - 1 \\ + \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} \phi \left(\frac{I_{t+1}^g}{I_t^g} - 1 \right) \left(\frac{I_{t+1}^g}{I_t^g} \right)^2 \end{array} \right\} \tag{29}$$

Competition will ensure zero profits for the goods-producing firms. Accordingly, the profits rebated to households as aliquot shares will be given by

$$\Pi_t = Q_t \left[1 - \frac{\phi}{2} \left(\frac{I_t^g}{I_{t-1}^g} - 1 \right)^2 \right] I_t^g - I_t^g. \tag{30}$$

3.4 The Government

The government finances its transfers T_t by issuing government bonds B_t^G to balance its budget period by period:

$$T_t = B_t^G - R_{t-1}^B B_{t-1}^G. \tag{31}$$

The household and firms can buy government bonds, so

$$B_t^G = B_t^H + D_t, \tag{32}$$

where D_t denotes firms' holdings of government bonds. We assume that the government will not make bonds available to households but only sell them to firms so that

$$B_t^H = 0 \tag{33}$$

and only use the households' stochastic discount factor to price the government bond. It implies that the government budget constraint can be written as:

$$T_t = D_t - r_{t-1}^B D_{t-1}. \tag{34}$$

Since firms that take the outside option are subject to a haircut cost ξ on their investment in government bonds in the previous period, the amount of transfers rebated to the household to ensure that there are no deadweight losses in the economy is equal to

$$\Xi_t = \xi D_{t-1}. \tag{35}$$

4 Analytical Characterization of the Equilibrium

In this section, we provide conditions that pin down the loan rate ρ_t and two cutoff points $\overline{\omega}_t$ and $\overline{\overline{\omega}}_t$. These cutoff points sort firms into three segments depending on the realization of the idiosyncratic productivity shock: 1) firms with $\omega \geq \overline{\omega}_t$ lend; 2) firms with $\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$ borrow and default; 3) firms with $\omega \geq \overline{\overline{\omega}}_t$ borrow and produce. Our strategy is to posit a solution that satisfies the first-order conditions of the intermediate goods sector and then verify that no firm can or has an incentive to switch to a different segment. To this purpose, we formulate three propositions.

4.1 The Loan Rate and Cutoff Points

Here we clarify the importance of the screening technology that allows lenders to tell whether borrowers can be expected to make more than the outside option. This technology prevents the firms with the lowest firm-specific technology levels from borrowing.

Assume (and then verify) that if the firms that have a firm-specific productivity $\omega < \overline{\omega}_t$ will choose to lend, then the following condition holds:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\overline{\omega}_t+\frac{(1-\delta)}{Q_t}Q_{t+1}\right)\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\right]. \tag{36}$$

This condition imposes that for a firm with productivity right at the cutoff point between borrowing and lending, $\omega = \overline{\omega}_t$, the expected return from producing $R_{t+1}(\overline{\omega}_t)$ equals the expected return of the outside option $R_t^B - \xi$.

We also need to impose an additional condition to induce a firm to lend. We need to compare the return from lending to the return from diverting funds. Firms that lend, will only lend as long as the return from lending, the lending rate net of the losses expected from firms diverting funds, matches the returns from borrowing and defaulting:

$$\begin{aligned}&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]=\\&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\omega}_t)b_t\right)\right].\end{aligned} \tag{37}$$

Notice there is no dependence of b_t on ω because $b_t = b_t(\omega)$ for all ω due to asymmetric information. At this point, we have spelled out all the elements that are needed for our first proposition.

Proposition 1. *Given that $\Theta_t(\omega)$ is non-negative and increasing in ω, firms with idiosyncratic productivity ω less than the cutoff point $\overline{\omega}_t$ will have no incentive to deviate from lending.*

A proof of this proposition is offered in Appendix A. The proof relies primarily on equations (36) and (37).

For the next step of pinning down $\overline{\overline{\omega}}_t$, consider the following condition:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\overline{\omega}}_t^*)b_t\right)\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\rho_tb_t\right)\right]. \tag{38}$$

It imposes that the marginal firm with productivity level $\overline{\overline{\omega}}_t^*$ will be indifferent between diverting funds and producing.

Taking the first derivative with respect to $\overline{\overline{\omega}}_t^*$ of both sides of equation (38), we will also need to impose the following condition on these derivatives:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\overline{\overline{\omega}}_t^*)b_t\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}'(\overline{\overline{\omega}}_t^*)(a_t+b_t)\right)\right]. \tag{39}$$

In other words, for a marginal firm with the productivity level $\overline{\overline{\omega}}_t^*$ the expected returns from diverting funds grow less steeply than the expected returns from producing. This condition is really an additional condition on the choice of the function $\Theta_t(\omega)$. It implies that a firm with a slightly higher productivity than $\overline{\overline{\omega}}_t^*$ will prefer producing to diverting funds, while a firm with a slightly lower productivity than $\overline{\overline{\omega}}_t^*$ will prefer diverting funds to producing. Note that it is a local condition in the sense that it applies to a marginal firm with the productivity level $\overline{\overline{\omega}}_t^*$.

We define $\overline{\overline{\omega}}_t=\max(\overline{\omega}_t,\overline{\overline{\omega}}_t^*)$. This definition ensures that terms that represent shares in the expression of the return from lending, for instance, $\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}$ in equation (37), are well-defined. But we can do more and establish that $\overline{\overline{\omega}}_t^*$ is never less than $\overline{\omega}_t$, in line with the following proposition.

Proposition 2. *Take $\overline{\overline{\omega}}_t^*$ to be the productivity of a firm indifferent between diverting funds and producing. Given that $\Theta_t(\omega)$ is non-negative, then it must be that $\overline{\overline{\omega}}_t^*\geq\overline{\omega}_t$.*

Notice that, by definition of $\overline{\overline{\omega}}_t$, a corollary of Proposition 2 is that $\overline{\overline{\omega}}_t=\overline{\overline{\omega}}_t^*$.

Before moving to our final proposition, we need to introduce one more condition on the function $\Theta(\omega)$. Assuming the function is convex, we need to choose a function such that

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(1)b_t\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\right)(a_t+b_t)\right]. \tag{40}$$

The left-hand side of the inequality is the maximum bound of the first derivative of the returns to borrowing and defaulting, whereas the right-hand side is the first derivative of the returns of firms that produce. Given that this condition holds for the most efficient firm with $\omega=1$ that has the largest marginal benefits from diversion, it will also hold for all firms with $\omega<1$ whose marginal benefit from diversion is smaller (including $\omega=\overline{\overline{\omega}}_t$). Accordingly, this global condition implies the local condition (39). We can use this global condition to prove that no diverting firm has an incentive to deviate by producing, and no producing firm has an incentive to switch to diverting. With this condition specified, we can state our final proposition.

Proposition 3. *Given that $\Theta_t(\omega)$ is non-negative, convex and increasing in ω, equations (36), (37), and (38), together with the slope condition under (40) are sufficient to ensure that, depending on the realization of their idiosyncratic productivity ω, firms sort themselves into three groups:*

1. *firms with $\omega \geq \overline{\omega}_t$ lend;*
2. *firms with $\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$ borrow and default;*
3. *firms with $\omega \geq \overline{\overline{\omega}}_t$ borrow and produce.*

A proof of this proposition is in Appendix A.

4.2 Aggregation and Equilibrium

We proceed as follows: First, we link individual and aggregate variables, then we define a competitive equilibrium.

Since a mass $\mu(\overline{\omega}_t)$ of firms lend and the complement mass $1-\mu(\overline{\omega}_t)$ of firms borrow, the inter-firm market clears when

$$\int_{0}^{\overline{\omega}_t} l_t(\omega)\mu'(\omega)\mathrm{d}\omega = \int_{\overline{\omega}_t}^{1} b_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{41}$$

which by defining

$$L_t = \int_{0}^{\overline{\omega}_t} l_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{42}$$

$$B_t = \int_{\overline{\omega}_t}^{1} b_t(\omega)\mu'(\omega)\mathrm{d}\omega \tag{43}$$

translates into:

$$L_t = B_t. \tag{44}$$

Since each type of firm borrows the same amount, we have $a_t(\omega) = a_t$ and $b_t(\omega) = b_t$ for each ω. Using the definition $A_t = \int_0^1 a_t(\omega)\mu'(\omega)\mathrm{d}\omega$ and equation (43), we can relate individual to aggregate variables as follows:

$$a_t = A_t, \tag{45}$$

$$b_t = \frac{B_t}{1-\mu(\overline{\omega}_t)}. \tag{46}$$

Next, consider the aggregation of budget constraints of firms in each segment. For lending firms, $l_t(\omega) = a_t(\omega)$ for each ω. Aggregating over the relevant mass of firms, i.e.,

$$\int_0^{\overline{\omega}_t} l_t(\omega)\mu'(\omega)\mathrm{d}\omega = \int_0^{\overline{\omega}_t} a_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{47}$$

results into

$$L_t = \mu(\overline{\omega}_t)A_t. \tag{48}$$

Let $d_t(\omega) = a_t(\omega) + \Theta_t(\omega)b_t(\omega)$ define the amount of resources diverted and invested in the outside option by the firm of productivity ω. Aggregating over the relevant mass of firms, i.e.,

$$\int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \left(a_t(\omega) + \Theta_t(\omega)b_t(\omega)\right)\mu'(\omega)\mathrm{d}\omega = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} d_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{49}$$

and combining it with the market-clearing condition in the bond's market, i.e.,

$$D_t = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} d_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{50}$$

result into

$$D_t = \left(\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)\right)A_t + \frac{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\theta B_t. \tag{51}$$

For producing firms, $a_t(\omega) + b_t(\omega) = b_t^{tot}(\omega)$. Aggregating over the relevant mass of firms, i.e.,

$$\int_{\overline{\overline{\omega}}_t}^{1} \left(a_t(\omega) + b_t(\omega)\right) \mu'(\omega)\mathrm{d}\omega = \int_{\overline{\overline{\omega}}_t}^{1} b_t^{tot}(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{52}$$

and defining

$$B_t^{tot} = \int_{\overline{\overline{\omega}}_t}^{1} b_t^{tot}(\omega)\mu'(\omega)\mathrm{d}\omega. \tag{53}$$

result into

$$B_t^{tot} = \left(1 - \mu(\overline{\overline{\omega}}_t)\right) A_t + \frac{1 - \mu(\overline{\overline{\omega}}_t)}{1 - \mu(\overline{\omega}_t)} B_t. \tag{54}$$

In equilibrium, all firms that produce will raise the same amount of financing. Therefore, $k_t(\omega) = \tilde{k}_t$ and, based on the properties of constant-returns-to-scale production functions, $h_{t+1}(\omega) = \tilde{h}_{t+1}$, where $\tilde{k}_t$ and $\tilde{h}_{t+1}$ are constant across firms. Accordingly, total production will be

$$Y_{t+1} = \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1}\omega \tilde{k}_t^{\alpha} \tilde{h}_{t+1}^{1-\alpha} \mu'(\omega)\mathrm{d}\omega, \tag{55}$$

such that the aggregate level of capital and labor are related to individual values as follows:

$$K_t = \int_{\overline{\overline{\omega}}_t}^{1} \tilde{k}_t \mu'(\omega)\mathrm{d}\omega = (1 - \mu(\overline{\overline{\omega}}_t))\tilde{k}_t, \tag{56}$$

$$H_{t+1} = \int_{\overline{\overline{\omega}}_t}^{1} \tilde{h}_{t+1} \mu'(\omega)\mathrm{d}\omega = (1 - \mu(\overline{\overline{\omega}}_t))\tilde{h}_{t+1}. \tag{57}$$

Therefore,

$$Y_{t+1} = \frac{1}{1 - \mu(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1}\omega K_t^{\alpha} H_{t+1}^{1-\alpha} \mu'(\omega)\mathrm{d}\omega. \tag{58}$$

Individual producing firms borrow to finance the purchase of capital, i.e., $b_t^{tot}(\omega) = Q_t k_t(\omega)$, so aggregating over this mass of firms results into the aggregate constraint

$$B_t^{tot} = Q_t K_t. \tag{59}$$

Let R_t define the average return on capital that producing firms receive. To this purpose, the relevant probability density function is $\frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})}$.

$$R_t = \int_{\overline{\overline{\omega}}_t}^{1} R_t(\omega)\frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})}\mathrm{d}\omega. \tag{60}$$

Substituting for $R_t(\omega)$ from equation (17):

$$R_t = \int_{\overline{\overline{\omega}}_t}^{1} \left(\frac{1}{Q_{t-1}}\alpha\ Z_t\left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha}\omega + \frac{(1-\delta)}{Q_{t-1}}Q_t\right)\frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})}\mathrm{d}\omega. \tag{61}$$

The aggregate equity return to households is defined as

$$R_t^A = \int_0^1 R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega \tag{62}$$

The Appendix shows that this equation can be used to get that

$$R_t^A A_{t-1} = B_{t-1}^{tot}R_t + (R_{t-1}^B - \xi)D_{t-1} + (1-\theta)\frac{\mu(\overline{\overline{\omega}}_{t-1}) - \mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}B_{t-1}. \tag{63}$$

This equation says that the funds paid to the household are equal to the returns from lending to producing firms net of intermediation costs, plus the returns from diverting funds, and including the funds that were lent to diverting firms and recovered.

The goods market clears:

$$Y_t = C_t + I_t^g. \tag{64}$$

Now we are ready to define a competitive equilibrium. The equilibrium is an allocation

$$\left\{C_t, H_t, Y_t, K_t, B_t^{tot}, I_t^n, I_t^g, \Pi_t, T_t, B_t^G, B_t^H, D_t, \Xi_t, \overline{\omega}_t, \overline{\overline{\omega}}_t, Z_t, FS_t, F_t, L_t, A_t, B_t\right\}_{t=0}^{\infty},$$

together with the sequence of prices $\left\{\lambda_{ct}, R_t^A, W_t, R_t^B, Q_t, R_t, \rho_t,\right\}_{t=0}^{\infty}$ satisfying equations (4), (5), (6), (7), (8), (10), (16), (26), (27), (29), (30), (31), (32), (33), (35), (36), (37), (38), (68), (44), (48), (51), (54), (58), (59), (61), (63), and (64), together with the the slope conditions defined in the Appendix, given initial conditions K_0, B_0^{tot}, B_0^G, B_0^H, D_0, $\overline{\omega}_0$, $\overline{\overline{\omega}}_0$, A_0, Z_0, FS_0, and the exogenous processes $\left\{\varepsilon_t^z, \varepsilon_t^f\right\}$.[13]

5 Parameter Choices and Model Solution

After reviewing the choice of key functional forms, we give a detailed account of the model parameterization.

5.1 Functional Forms

We choose the Beta distribution to governs draws of the idiosyncratic productivity ω. With parameters η_1 and η_2, the probability density function is

$$\mu'_{\eta_1,\eta_2}(\omega) = \frac{\omega^{\eta_1 - 1}(1-\omega)^{\eta_2 - 1}}{B(\eta_1, \eta_2)}, \tag{65}$$

where

$$B(\eta_1, \eta_2) = \frac{\Gamma(\eta_1)\Gamma(\eta_2)}{\Gamma(\eta_1 + \eta_2)} \tag{66}$$

and Γ is the Gamma function. This distribution affords us the flexibility of large differences in productivity across firms that choose to produce and borrow, while capturing that financial intermediaries account for a large fraction of the financing available to firms that produce.

We also need to take a stand on the function $\Theta_t(\omega)$, which governs the fraction of the funds borrowed from other firms $b_t(\omega)$. We pick

$$\Theta_t(\omega) = \omega^{\psi} F_t, \tag{67}$$

where ψ is a parameter, and F_t is simply introduced to ensure that equation (11) holds.

$$\theta = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \omega^{\psi} F_t \frac{\mu'(\omega)}{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)} \mathrm{d}\omega. \tag{68}$$

As we have seen, the choice of functional form for $\Theta_t(\omega)$ can be used to ensure the slope

[13] To express equations (37) and (38) in terms of the aggregate variables A_t, B_t, and F_t, we use equations (45), (46), and (67) to substitute for a_t, b_t, and $\Theta_t(\omega)$, respectively.

condition in (40), which in turn is used in our proof that the idiosyncratic productivity ω will determine whether any firm is a borrower or a lender. We resort to numerical simulations to check this slope condition numerically for any set of shocks. When setting $\psi = 2$ we have not encountered any cases that violated the slope condition.

5.2 Parameters

We choose model parameters through estimation, calibration, and by reference to other papers in the literature. Specifically, we estimate model-specific parameters using the simulated method of moments (SMM). We calibrate others to match steady-state targets derived from long-run averages. Finally, we set a few parameters to standard values from the literature.

We use the simulated method of moments (SMM) to size the shock processes for total factor productivity, Z_t, and financial shock, FS_t. We model each shock as an AR(1) process. We size the persistence parameters and standard deviations of the innovations, along with the investment adjustment cost parameter, ϕ.

For the implementation of the SMM estimation, we minimize a quadratic objective function based on variances, correlations, and autocorrelations of real GDP, real consumption, the relative price of investment, and the average delinquency rate on business loans at commercial banks.[14] We should stress that in our simple model there is no distinction between delinquency rates, charge-off rates, and the outright default rate of borrowers.

Our data run from the first quarter of 1987 through the fourth quarter of 2021, matching the sample period in Section 2, a choice dictated by the availability of data on the dispersion of productivity. The data moments for the SMM exercise are computed after HP-filtering the data (using a value for the smoothing parameter of 1,600 as standard for quarterly data). We take the same approach for observed and simulated data. For the SMM objective function, we employ a modified optimal weighting matrix with model moments from a 10000-

[14] We use chain-type indexes for relative prices and GDP. The relative price of investment is the ratio of the price index for gross private domestic investment to the price index for personal consumption expenditures excluding food and energy. These price indexes and GDP are from the National Income and Product Accounts of the U.S. Bureau of Economic Analysis. The delinquency rate on business loans at commercial banks is from the data release Charge-Off and Delinquency Rates on Loans and Leases at Commercial Banks of the Board of Governors of the Federal Reserve System. The series for real GDP and delinquency rates are the same as those used in Section 2.

observation simulated sample. Specifically, we replace the weight on the real GDP-default rate correlation with the matrix's maximum diagonal element, the weights on the investment price variance and default rate variance each with half that maximum diagonal element.

We calibrate selected parameters to match steady-state targets for key variables, using first moments from the same sample period as the SMM estimation. We size the two parameters of the Beta distribution η_1 and η_2, the haircut ξ, and the diversion share θ at values that jointly match the 1) within-industry dispersion of productivity as captured by the weighted average gap in TFP between the 90^{th} and 10^{th} percentile firm, 2) the average spread between the loan rate and interbank lending rate, 3) the average share of bank credit, and 4) the average delinquency rate on business loans.[15] The idea behind these choices of the parameters is the following. The shape of the productivity distribution is important to match that the establishment at the 90^{th} percentile is 2.08 as productive as the one at the 10^{th} percentile. We compute the target ratio of weighted-average within-industry dispersion over the sample period by taking the mean of the data points underlying the solid line in the top panel of Figure 1. Appendix E provides formulas to calculate this target ratio from the model. We map lending firms into stylized banks. The parameter ξ is relevant for matching that banks provide about 47% of private credit in the United States.[16] We equate this value to our model counterpart expression $\frac{L_t}{B_t^{tot}}$ evaluated in the steady state. We use the same data on delinquent loans as in Section 2 to size the average delinquency rate at 2.6%. We map it to the share of borrowing firms diverting funds $\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}$ to calibrate θ. We calculate the average spread between the bank prime loan rate and the 3-month interbank rate at 2.7% annualized and map it to the difference between the average return on capital and the loan rate in steady state of the model, i.e., $R-\rho$, to further validate the model's fit to financial variables.[17]

Before discussing SMM-estimated parameters, we first outline the calibration of the re-

[15]The shape of the productivity distribution affects all the described targets, so the word "*jointly*" follows. For exact identification, we consider four variables to match four targets.

[16]We construct our measure using data from the Z.1 Financial Accounts of the United States. We sum non-financial corporate and non-financial non-corporate business loans, subtract non-financial business other loans and advances, divide by total credit to non-financial corporations from the Bank for International Settlements, and take the average.

[17]Bank prime loan rate data is reported in the Federal Reserve Board's H.15 Release, and 3-month interbank rate data are from the Organization for Economic Co-operation and Development.

maining parameters. Calibrated values appear at the top of Table (4) and follow guidance from prior literature: the discount rate $\beta = 0.9925$, the capital share $\alpha = 0.3$, the depreciation rate $\delta = 1$, the inverse Frisch elasticity of labor supply $\nu = 2$, and the inverse intertemporal elasticity of substitution $\sigma = 1.1$ that approximates the log case.

We set ϑ at 0.83, a value that ensures that aggregate labor takes a value of 1 in the steady state.

5.3 Simulated Method of Moments, Results

As shown in Table 5, the second moments targeted in the SMM-estimattion are remarkably close to their data counterparts. The table also reports the 5^{th} and 95^{th} percentiles from 1000 simulated samples of the same length as the observed data. All the model-estimated correlations and autocorrelations have the same signs as in the data. The variances are matched especially well.

Table 6 reports the relative contributions of the shocks in our calibration to fluctuations in variables targeted by the SMM procedure and other key variables. Aggregate TFP shocks are the primary drivers of fluctuations in GDP and TFP dispersion, while the interest rate shock, our financial shock, dominates fluctuations in consumption. The two shocks have comparable importance for the variation in investment, default rate, and short-term real interest rates.

6 Dynamic Responses to Shocks and Model Assessment

Before turning to an assessment of the model by checking untargeted moments, Section 6.1 discusses the effects of an aggregate TFP shock. Section 6.2 presents the impulse response functions to a financial shock that tightens financial conditions. We compare our model to a frictionless RBC model, a special case of our model in which the most efficient is the only one to produce, managing to attract all household savings.

6.1 An Expansionary TFP Shock

Figure 2 shows the responses of key aggregate variables to a one-standard-deviation expansionary TFP shock, ε_t^z. The size of the shock process and its persistence follow the estimates from Section 5. In each panel, the solid lines show responses from the baseline model with financial frictions and endogenous TFP dispersion. The dashed lines show responses in a frictionless RBC model, a special case of our model when all production is carried out by the most efficient firm.

On impact (period 1 in the Figure), the positive productivity shock increases the marginal product of capital, which in turn raises the average return on capital. It is this standard exogenous productivity that pushes up aggregate output on impact. An endogenous TFP channel kicks in from the second period.

Since the shock is autocorrelated, today's TFP shock reduces the marginal product of capital expected for next-period. In line with this effect, the demand for capital after initially expanding starts declining. This expected decline brings down the expected price of capital. Accordingly, the average return to capital falls. This return is averaged across firms with different idiosyncratic productivity, and it includes a term that reflects the marginal product of capital and another term that captures gains or losses from the resale of undepreciated capital. It is this second term that pushes down the average return on capital.

As a reminder, Equation (38) pins down $\overline{\overline{\omega}}_t$ From this equation, we can see that, all else equal, a decrease in the average return of capital pushes up the $\overline{\overline{\omega}}_t$. While many of the variables entering equation (38) do change, the decrease in the average return of capital is the most important quantitatively. Intuitively, more productive firms that are better able to withstand a decrease in the average return to capital. As $\overline{\overline{\omega}}_t$ rises, the productivity dispersion declines, as captured by a reduction in the gap between the productivity of the 90^{th} and 10^{th} percentile firms.

As the aggregate technology shock is expected to gradually wane, the the risk-free interest rate declines. This decline makes the outside option less attractive. Accordingly, the default rate goes down. In turn, the returns to lending increase. As more firms find it attractive to lend, the productivity of a firm indifferent between lending and borrowing, $\overline{\omega}$, increases.

Notice that as $\overline{\overline{\omega}}_t$ does not rise quite as much as $\overline{\omega}_t$, the default rate goes down. Accordingly, default rates move countercyclically. However, it turns out that financial shocks play an even larger role than technology shock in driving the default rate through the lens of our model. We turn to financial shocks next.

Due to a fall in the productivity dispersion, investment rises more than in a frictionless RBC model. Total output also increases more. On impact, consumption rises relatively less compared to a frictionless RBC model as consumption is crowded out by more productive investments. Subsequently, starting from the next period, consumption rises relatively more due to higher total output. Moreover, endogenous productivity dispersion generates a hump-shaped response of investment.

6.2 A Contractionary Financial Shock

Here we study a negative ε_t^f innovation to the shock FS_t in the household's Euler condition in Equation (5). We size the innovation at one standard deviation and choose its persistence, based on the estimates presented in Section 5. Figure 3 shows the effects of this shock.

The shock increases the cost of borrowing. The risk-free rate goes up and so does the inter-firm lending rate, ρ_t. Remembering, again that Equation (38) pins down $\overline{\overline{\omega}}_t$, we can see that an increases in the risk-free rate and the lending rate would push up $\overline{\overline{\omega}}_t$, all else equal. Intuitively, only the more productive firms can withstand to pay a higher lending rate without defaulting. The effects are extremely persistent. This persistence results from the high persistence of the shock process interacting with the slow-moving capital accumulation process. The initial increase in the risk free rate spikes because of investment adjustment costs, which reduce the short-run interest sensitivity of the model economy. But the initial spike masks that even after 20 quarters, the risk-free rate has not quite settled back down to its steady state level. As a result the small increase in $\overline{\overline{\omega}}_t$ can be extremely persistent. The response of the frictionless economy is equally persistent, underlying that the exogenous persistent of the shock is doing much of the work here.

Where the frictionless model is substantially different is in the response of total output. As the financial shock leads to a persistent effect on $\overline{\overline{\omega}}_t$, lower productivity firms stop producing, the endogenous component of TFP expands, and total output increases — the opposite as for the frictionless model without and endogenous TFP component. In that model, the lower investment reduces the capital stock, which brings output down when it is not offset by the endogenous response of TFP.

The effects of the shock on $\overline{\omega}_t$ are governed by forces that move in opposite directions. Upfront, the increase in the risk-free rate dominates. All else equal, this increase makes the outside option more attractive, accordingly the returns to lending fall, and $\overline{\omega}_t$ declines. But another force eventually shines through as the risk-free rate starts coming back down. The increase in $\overline{\omega}_t$ means the net return to lending is pushed up, which makes lending more attractive, all else equal. This effect eventually dominates and $\overline{\omega}_t$ moves up.

The default rate is just governed by the difference in the response of $\overline{\overline{\omega}}_t$ and $\overline{\omega}_t$ It rises persistently, but the rise is more prominent upfront.

We have verified that the model is stationary despite the high persistence of the effects of the financial shock. Extending the responses to the shock beyond the 20 quarters shown in the figure, the responses do eventually die down, with all variables returning to their steady-state values.

6.3 Untargeted Moments

Having illustrated the response of the model to technology and financial shocks individually, we can now turn to an assessment of unconditional moments. To check whether the model is consistent with the regression results discussed in Section 2, we draw 1,000 data samples with 500 quarters in each sample. We draw longer synthetic samples than the observed data to compensate for the missin panel dimension. For each of the 1,000 samples, we estimate regressions analogous to those of Section 2.[18] Table 7 compares slope coefficients between model-simulated data (averaged over 1,000 samples) and observed data, including 95% confidence intervals ($2.5^{th} - 97.5^{th}$ percentiles for simulations; standard errors for data).

As shown in panels (a) and (b) of Table 7, the model-based and data-based confidence intervals overlap for each regression. Thus, we cannot reject the hypothesis that the estimated model and observed data yield statistically indistinguishable slope coefficients on the default

[18]Since the model is calibrated at quarterly frequency while observed data are annual, we annualize model-simulated data following standard practices.

rate and real interest rate at the 5% significance level. Panel (c) shows that the model-based slope coefficient on the GDP growth rate is negative at any reasonable significance level and has the same sign as in the empirical regression. However, the confidence intervals do not overlap. This difference can result from the fact that there is a trend component of real GDP in the data, affecting the calculation of the growth rate, while model-simulated data are stationary. To control for this possibility, we extract cyclical components from the 90-10 TFP dispersion and real GDP with the HP filter using a coefficient of 6.25. Panel (d) shows the regression results from this procedure. Since the confidence intervals overlap, we cannot reject the hypothesis that the estimated model and observed data produce statistically indistinguishable slope coefficients at the 5% significance level for the regression of cyclical TFP dispersion on cyclical real GDP.

7 Conclusion

We have developed a model in which informational frictions give rise to moral hazard and credit misallocation. In our model, as in Stiglitz and Weiss (1981), credit is rationed.

Firms are subject to idiosyncratic productivity shocks that are private information. Households cannot get a good read on the productivity of different firms and end financing them all. Depending on their productivity draw, firms sort themselves into lenders, essentially becoming financial intermediaries, and borrowers. Borrowers are subject to a default decision.

We show that this tractable model can capture key facets of the data. First and foremost, it is able to capture the average within-industry dispersion in productivity observed at the plant level in the United States. This result implies that financial frictions can explain why firms with substantial productivity differences coexist. The model also does a good job in matching the cyclical variation in dispersion and its correlation with GDP and short-term real interest rates, despite the fact that these moments are not directly targeted.

These changes in productivity dispersion imply that component of total factor productivity evolves endogenously. We have shown that a tightening in financial conditions forces unproductive firms to quit and raises total factor productivity. We have also shown that this endogenous component of productivity can induce notable TFP movements for realistically sized shocks.

We hope that extension of our tractable model can serve as a building block in the exploration of how alternative policies affect productivity. Extensions could allow the study of fiscal and monetary policy choices. Going beyond our current model, we can speculate that expansionary fiscal policies could have additional desirable effects in our model as the associated increases in real interest rates would force some unproductive firms to quit. By contrast, monetary policy would face additional challenges in our setup. The typical New Keynesian rationale to lower policy rates in a downturn of aggregate demand could inefficiently keep low-productivity firms afloat. In sum, we see our work as opening promising avenues for further research.

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Tables and Figures

Table 2: Panel regression of within-industry TFP dispersion on (1) GDP growth, (2) delinquency rate on business loans at commercial banks, and (3) Real interest rate — 1987-2021

	Percentage difference between the 90th and 10th percentile of the log of TFP		
	(1)	(2)	(3)
Yearly GDP growth (%)	-2.875*** (-6.397)		
Delinquency rate (%)		-3.744*** (-3.726)	
Real interest rate (%)			-3.730*** (-4.205)
Observations	3010	3010	3010
S.E. type	by: industry	by: industry	by: industry
R^2	0.618	0.626	0.635
R^2 Within	0.033	0.052	0.076

Significance levels: $*$ p $<$ 0.05, $**$ p $<$ 0.01, $***$ p $<$ 0.001. In the table, t-stats are in parentheses. All regressions are weighted using industry shares in value added and include industry fixed effects.

Table 3: Robustness of panel regression of within-industry TFP dispersion on (1) GDP growth, (2) delinquency rate on business loans at commercial banks, and (3) real interest rate — 1987-2021

	Percentage difference between the 75th and 25th percentile of the log of TFP		
	(1)	(2)	(3)
Yearly GDP growth (%)	-0.917*** (-4.774)		
Delinquency rate (%)		-1.057* (-2.218)	
Real interest rate (%)			-0.968* (-2.634)
Observations	3010	3010	3010
S.E. type	by: industry	by: industry	by: industry
R^2	0.671	0.672	0.673
R^2 Within	0.015	0.018	0.022

Significance levels: $*$ $p < 0.05$, $**$ $p < 0.01$, $***$ $p < 0.001$. In the table, t-stats are in parentheses. All regressions are weighted using industry shares in value added and include industry fixed effects.

Table 4: Model parameters

	Value	Description	
Conventional			
β	0.9925	Discount rate	
α	0.3	Capital share in production	
σ	1.1	Inverse intertemporal elasticity of substitution	
δ	0.01	Depreciation rate	
ν	2	Inverse Frisch elasticity of labor supply	
Calibrated (first-order moments matching steady-state conditions)			*Targets/Explanation*
η_1	1.71053	First parameter of Beta distribution	jointly to match the average 1) 90-10 within-industry productivity dispersion, 2) spread between the loan rate and interbank lending rate, 3) share of bank credit, and 4) default rate on business loans
η_2	3.36804	Second parameter of Beta distribution	
ξ	0.00687	Haircut on the returns from the outside option	
θ	0.00031	Average fraction of funds that can be diverted	
Estimated (Simulated methods of moments)			*Targets/Explanation*
ϕ	0.20129	Investment adjustment costs	estimated to hit the variances, correlations, and autocorrelations of 1) real output, 2) real consumption, 3) relative investment price, and 4) default rate
ρ_z	0.50065	Persistence of technology shock	
ρ_f	0.98767	Persistence of financial shock	
ε^z	0.00895	1 s.d. of technology shock	
ε^f	0.03402	1 s.d of financial shock	
Specific			*Explanation*
ψ	2	Parameter in the function $\Theta_t(\omega) = \omega^{\psi} F_t$	Ensures that the slope condition is verified
ϑ	0.83340	Disutility of labor	Supports aggregate labor = 1 in the steady state

Table 5: SMM estimation: Data and Model counterparts, 1987:Q1–2021:Q4

	Data	Model	Model 5th perc.	Model 95th perc.
Var(GDP)	1.70	2.28	1.54	2.92
Corr(GDP,Consumption)	0.91	0.59	0.41	0.73
Corr(GDP,Investment price)	0.31	0.67	0.60	0.74
Corr(GDP,Default rate)	-0.58	-0.60	-0.70	-0.51
Var(Consumption)	2.10	1.90	1.24	2.49
Corr(Consumption,Investment price)	0.22	0.30	0.16	0.44
Corr(Consumption,Default rate)	-0.42	-0.19	-0.32	-0.05
Var(Investment price)	0.56	0.62	0.47	0.75
Corr(Investment price,Default rate)	-0.48	-0.96	-0.97	-0.94
Var(Default rate)	0.34	0.27	0.21	0.34
Autocorr(GDP)	0.64	0.64	0.53	0.71
Autocorr(Consumption)	0.57	0.72	0.59	0.78
Autocorr(Investment price)	0.90	0.27	0.13	0.39
Autocorr(Default rate)	0.95	0.25	0.11	0.37

Note: The table reports the second moments targeted in the SMM procedure for the model calibration —variances, correlations, and autocorrelations at business-cycle frequencies. The model moments are computed from a simulated sample of 10,000 observations. Both observable and model-simulated data are HP-filtered using a standard value of 1,600 for the smoothing parameter. A modified optimal weighting matrix was used: the real GDP-default rate correlation receives the weight of the matrix's maximum diagonal element, while each of the investment price variance and default rate variance receives half of that maximum diagonal element. The last two columns show the 5^{th} and 95^{th} percentiles from 1,000 simulated samples of the same length as the observed data.

Table 6: Variance decomposition, 1987:Q1–2021:Q4

	Output	Investment	Consumption	Default Rate	Real Rate	90-10 TFP dispersion
TFP	99.76	49.05	34.50	42.79	51.67	93.79
Financial shock	0.24	50.95	65.50	57.21	48.33	6.21

The table shows the variance decompositions, in percent. TFP refers to the shock to total factor productivity, Z_t; Financial shock refers to the shock to borrowing conditions, FS_t,

Table 7: Model-simulated and observed data: regression results

(a)

	90-10 TFP dispersion	
	Model	Data
Default rate	-10.315***	-3.744***
	(3.084)	(1.005)
95% Confidence Interval	$[-15.761; -3.578]$	$[-5.713; -1.775]$

(b)

	90-10 TFP dispersion	
	Model	Data
Real interest rate	-0.999	-4.315***
	(1.359)	(1.096)
95% Confidence Interval	$[-4.507; 0.771]$	$[-6.463; -2.167]$

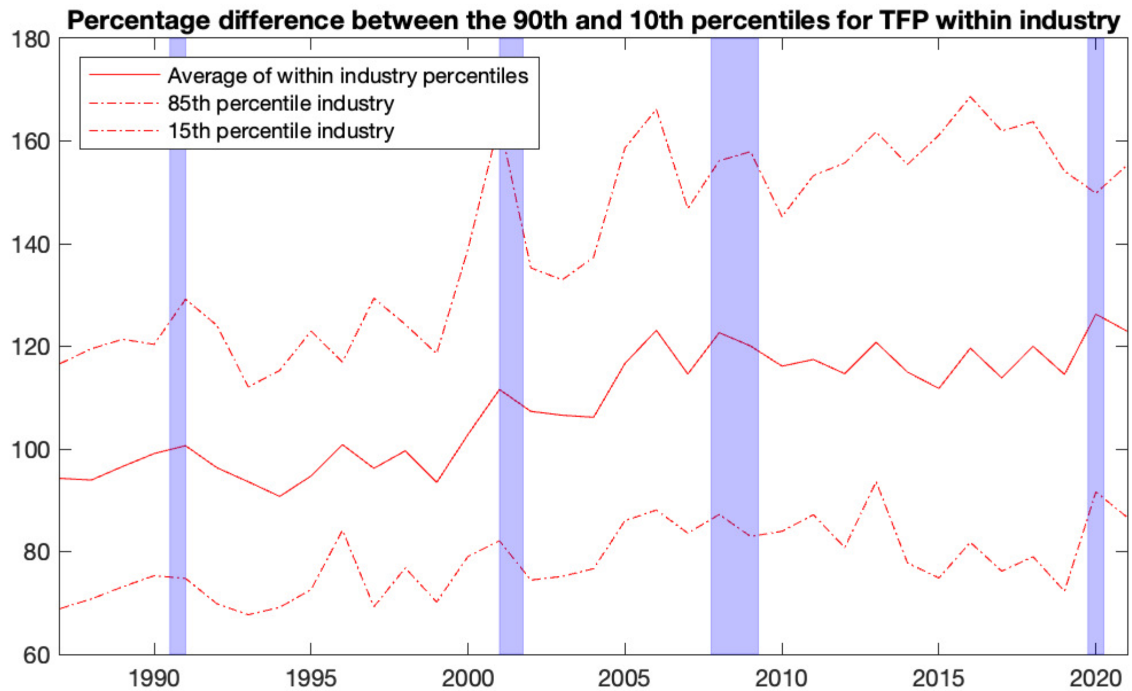
(c)

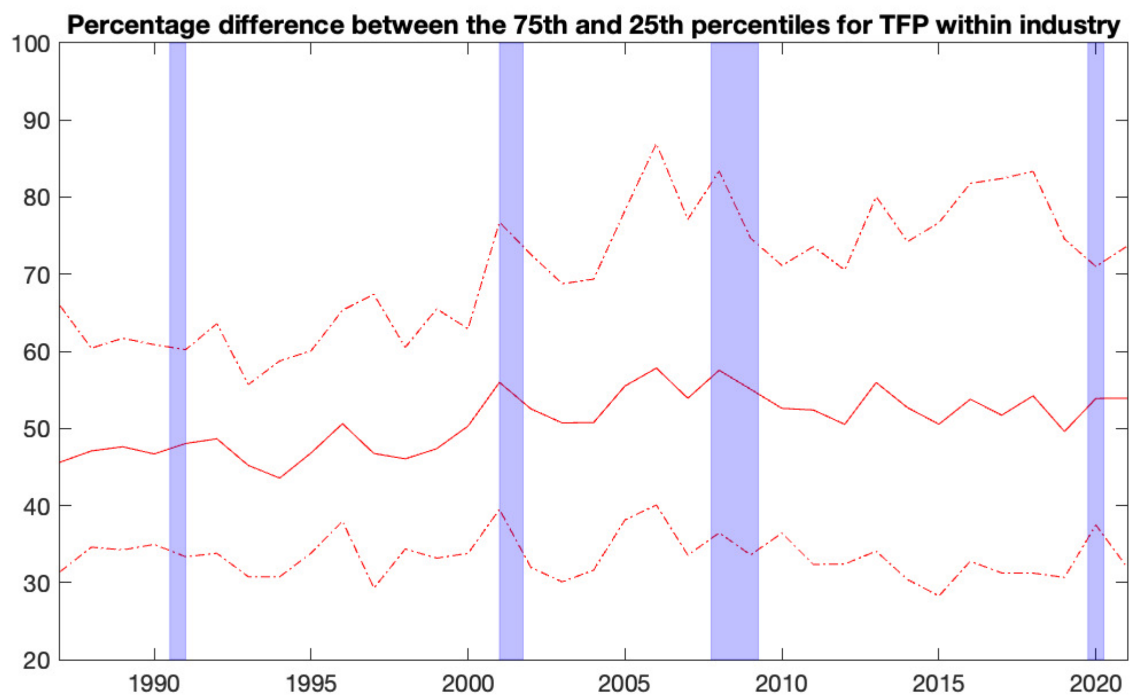
	90-10 TFP dispersion	
	Model	Data
GDP growth	-0.684***	-2.875***
	(0.101)	(0.449)
95% Confidence Interval	$[-0.926; -0.504]$	$[-3.756; -1.994]$

(d)

	90-10 TFP dispersion (cyclical)	
	Model	Data
Real GDP (cyclical)	-1.046***	-0.601*
	(0.091)	(0.345)
95% Confidence Interval	$[1.256; -0.880]$	$[-1.277; 0.075]$

Figure 1: TFP dispersion





Note: Shaded areas denote recessions as dated by the NBER Business Cycle Dating Committee.

Figure 2: A positive TFP shock

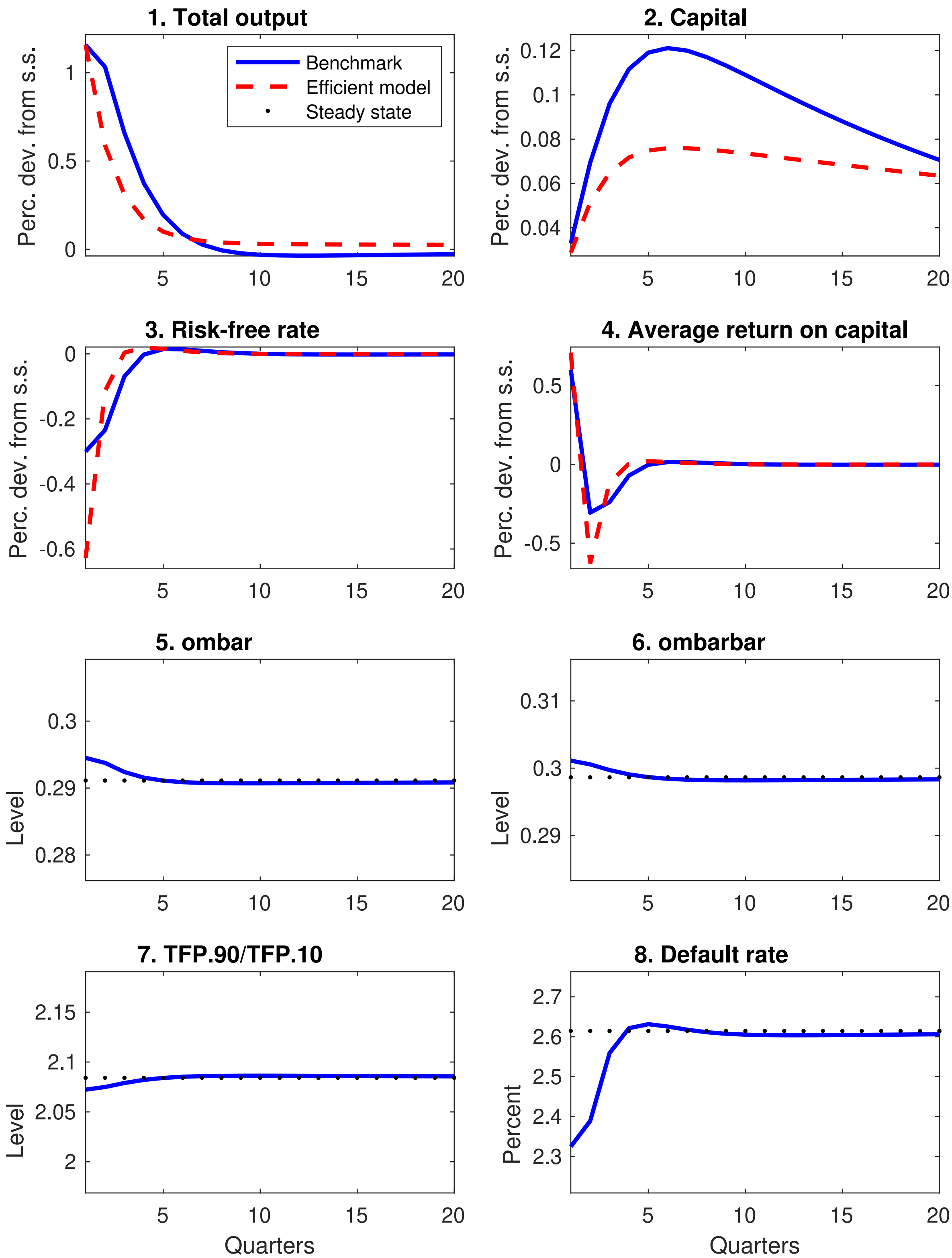
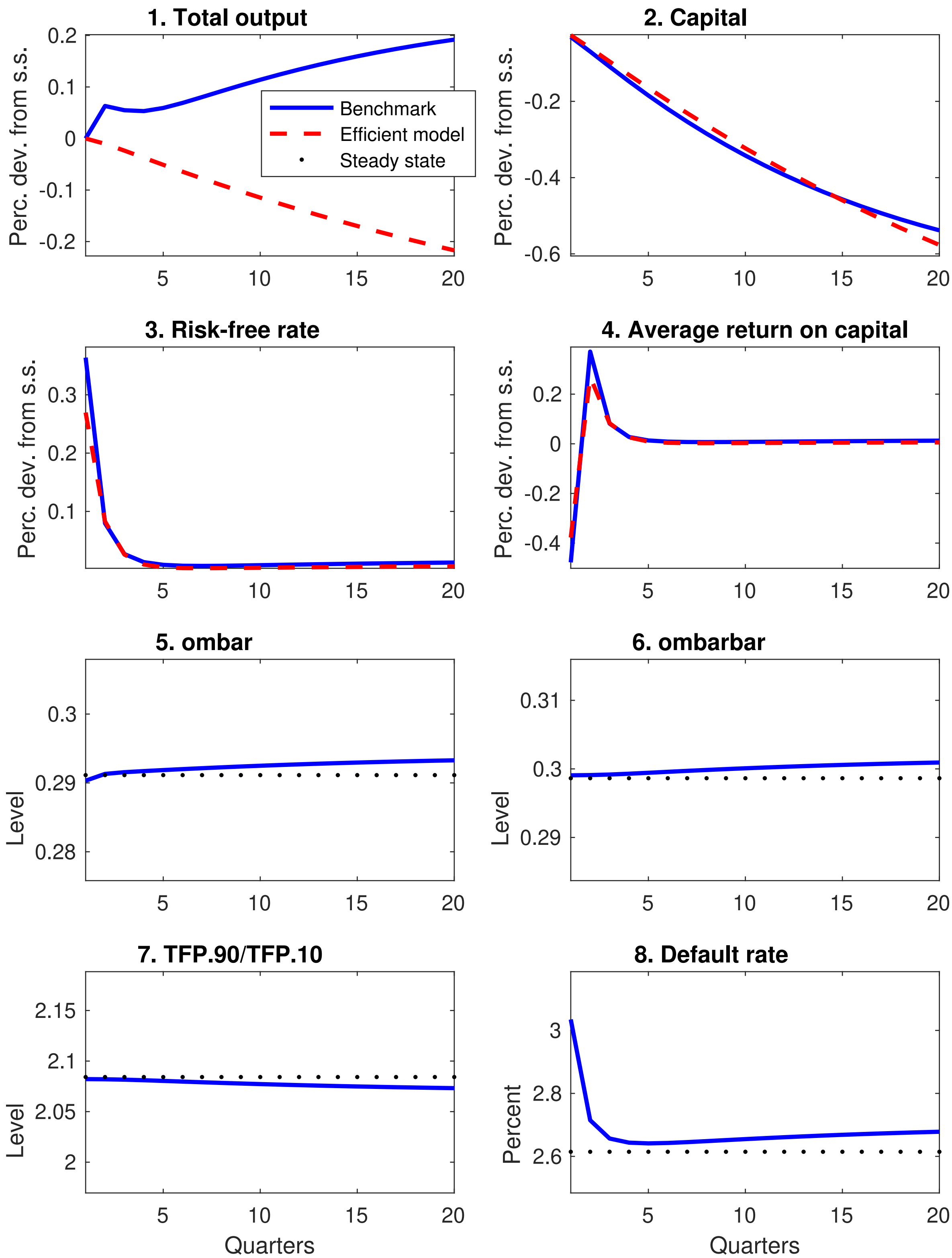


Figure 3: A negative financial shock



Online Appendix

A Proofs of Propositions

A.1 Proof of Proposition 1

Equation (37) can be described as

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]= E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\omega)b_t\right)\right]. \tag{A.1}$$

when $\omega=\overline{\omega}_t$. The left-hand side of equation (A.1) does not depend on ω, while the right-hand side of equation (A.1) increases in ω due to our assumption that $\Theta_t(\omega)$ is increasing in ω. Therefore, firms with $\omega<\overline{\omega}_t$ will have no incentive to deviate to the outside option because they get higher expected profits from lending than from diverting funds.

It is left to show that firms with $\omega<\overline{\omega}_t$ will have no incentive to deviate to production. We need to establish that the expected profits from lending are higher than the expected profits from producing, i.e.,

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]>E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right], \tag{A.2}$$

for all firms with $\omega<\overline{\omega}_t$. Notice that these firms cannot borrow in the inter-firm market because of the screening technology.

By combining equations (17) and (36), we get

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\right]. \tag{A.3}$$

Plug this result into equation (37):

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]= E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\left(a_t+\Theta_t(\overline{\omega}_t)b_t\right)\right]. \tag{A.4}$$

Notice that

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})a_t\right] > E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right] \tag{A.5}$$

for all firms with $\omega < \overline{\omega}_t$ since $R_{t+1}(\omega)$ is increasing in ω. Combining this result with $\Theta_t(\overline{\omega}_t)b_t \geq 0$ and $\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega}) \geq 0$, we get

$$\begin{aligned}E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)} + (1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right] = \\ E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\left(a_t + \Theta_t(\overline{\omega}_t)b_t\right)\right] \geq \\ E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})a_t + \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\Theta_t(\overline{\omega}_t)b_t\right] > E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right],\end{aligned} \tag{A.6}$$

for all firms with $\omega < \overline{\omega}_t$. Thus, equation (A.2) is verified. □

A.2 Proof of Proposition 2

Let us prove it by contradiction. Assume that $\overline{\overline{\omega}}^*_t < \overline{\omega}_t$. Therefore, by our definition $\overline{\overline{\omega}}_t = \max(\overline{\omega}_t, \overline{\overline{\omega}}^*_t) = \overline{\omega}_t$. Plugging this result into equation (37):

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\rho_t a_t\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R^B_t - \xi\right)\left(a_t + \Theta_t(\overline{\omega}_t)b_t\right)\right]. \tag{A.7}$$

Consider the right-hand-side of equation (38) and substitute for $E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\rho_t\right]$ from equation (A.7).

$$\begin{aligned}E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}^*_t)(a_t+b_t) - \rho_t b_t\right)\right] = \\ E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}^*_t)(a_t+b_t) - \left(R^B_t - \xi\right)\left(1 + \Theta_t(\overline{\omega}_t)\frac{b_t}{a_t}\right)b_t\right)\right].\end{aligned} \tag{A.8}$$

Using that $R_{t+1}(\omega) > 0$ and strictly increasing in ω, together with equation (A.3) lead to the following inequality:

$$0 < E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\overline{\omega}}^*_t)\right] < E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega}_t)\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R^B_t - \xi\right)\right] \tag{A.9}$$

Plugging this inequality into equation (A.8) and collecting terms result into:

$$
\begin{aligned}
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\rho_t b_t\right)\right]=\\
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\left(R_t^B-\xi\right)\left(1+\Theta_t(\overline{\omega}_t)\frac{b_t}{a_t}\right)b_t\right)\right]<\\
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(R_t^B-\xi\right)(a_t+b_t)-\left(R_t^B-\xi\right)\left(1+\Theta_t(\overline{\omega}_t)\frac{b_t}{a_t}\right)b_t\right)\right]= \qquad \text{(A.10)}\\
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(R_t^B-\xi\right)\left(a_t-\Theta_t(\overline{\omega}_t)\frac{b_t^2}{a_t}\right)\right)\right]\leq E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(R_t^B-\xi\right)a_t\right)\right]\leq\\
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\overline{\omega}}_t^*)b_t\right)\right],
\end{aligned}
$$

where we use that $\Theta_t(\overline{\omega}_t)\frac{b_t^2}{a_t}\geq 0$ and $\Theta_t(\overline{\overline{\omega}}_t^*)b_t\geq 0$. Therefore,

$$
E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\overline{\omega}}_t^*)b_t\right)\right]>E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\rho_t b_t\right)\right]. \qquad \text{(A.11)}
$$

However, inequality (A.11) contradicts equation (38). Therefore, $\overline{\overline{\omega}}_t^*>\overline{\omega}_t$ □

A.3 Proof of Proposition 3

Given the results of Propositions 1 and 2, we are left to show that

1. No firm with idiosyncratic productivity $\overline{\omega}_t<\omega<\overline{\overline{\omega}}_t$ has an incentive to deviate from borrowing and defaulting.
2. No firm with idiosyncratic productivity $\omega\geq\overline{\overline{\omega}}_t$ has an incentive to deviate from borrowing and producing.

We will show our proof in three smaller steps.

Step 1: Firms with $\omega>\overline{\omega}_t$ have higher expected profits from borrowing and defaulting than from lending.

Proof of step 1: Equation (37) can be described as

$$\begin{aligned} &E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]= \\ &E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)(a_t+\Theta_t(\omega)b_t)\right]. \end{aligned} \tag{A.12}$$

when $\omega=\overline{\omega}_t$. The left-hand side of equation (A.12) does not depend on ω, while the right-hand side of equation (A.12) increases in ω due to our assumption that $\Theta_t(\omega)$ is increasing in ω. Therefore, firms with $\omega>\overline{\omega}_t$ get higher expected profits from diverting funds than from lending. □

Step 2: Firms with $\omega<\overline{\overline{\omega}}_t$ have lower expected profits from borrowing and defaulting than from producing.

Proof of step 2:

Equation (37) can be described as

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)(a_t+\Theta_t(\omega)b_t)\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)(a_t+b_t)-\rho_tb_t\right)\right]. \tag{A.13}$$

when $\omega=\overline{\overline{\omega}}_t$. Taking the first derivative of both sides of equation (A.13) with respect to ω, we need to show that

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}'(\omega)(a_t+b_t)\right)\right] \tag{A.14}$$

for all $\omega\geq\overline{\overline{\omega}}_t$. According to this inequality, the expected profits from diverting funds grow less steeply than the expected profits from producing for all firms with the productivity level $\omega\geq\overline{\overline{\omega}}_t$. Together with the equalization of expected profits in equation (A.13) evaluated at $\omega=\overline{\overline{\omega}}_t$, this inequality implies that the expected profits from diverting funds are lower than the expected profits from producing for all $\omega<\overline{\overline{\omega}}_t$.

Let us show that condition (40) is sufficient to ensure inequality (A.14) for all $\omega\geq\overline{\overline{\omega}}_t$. Finding the derivative of $R_{t+1}'(\omega)$ from equation (17) and plugging it into equation (A.14), we get

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\right)(a_t+b_t)\right]. \tag{A.15}$$

Since $\Theta_t(\omega)$ is convex (so $\Theta_t'(\omega)$ is non-decreasing in ω), then

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right]\leq E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(1)b_t\right] \tag{A.16}$$

for all $\overline{\overline{\omega}}_t\leq\omega\leq 1$. Therefore, if

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(1)b_t\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\right)(a_t+b_t)\right] \tag{A.17}$$

holds, then it implies

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}'(\omega)(a_t+b_t)\right)\right] \tag{A.18}$$

for all $\omega\geq\overline{\overline{\omega}}_t$. It establishes the sufficiency of the global slope condition in equation (40). □

Step 3: Firms with $\omega>\overline{\overline{\omega}}_t$ have higher expected profits from producing than from lending.
Proof of step 3: It directly follows from the result of Proposition 2 and the two results shown in steps 1 and 2. Since $\overline{\overline{\omega}}_t\geq\overline{\omega}_t$, the result of step 1 implies that firms with $\omega>\overline{\overline{\omega}}_t$ have higher expected profits from diverting funds than from lending. Combining this implication with the result of step 2 that establishes that firms with $\omega>\overline{\overline{\omega}}_t$ have higher expected profits from producing than from diverting funds, ensures that firms with $\omega>\overline{\overline{\omega}}_t$ have higher expected profits from producing than from lending. □

Note that equations (37) and (38) are constructed such that a marginal firm with productivity level $\overline{\omega}_t$ is indifferent between lending and diverting funds, while a marginal firm with productivity level $\overline{\overline{\omega}}_t$ is indifferent between diverting funds and producing. For this proposition, we resolve the tie by assuming these marginal firms choose to lend and produce, respectively. Since the probability distribution of ω is continuous, the probability that $\omega=\overline{\omega}_t$ or $\omega=\overline{\overline{\omega}}_t$ is zero. □

B Derivations

B.1 Aggregate Equity Return

By definition

$$R_t^A = \int_0^{\overline{\omega}_{t-1}} R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega + \int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}} R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega + \int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega =$$

$$\int_0^{\overline{\omega}_{t-1}} \left(\rho_{t-1}\frac{1-\mu(\overline{\overline{\omega}}_{t-1})}{1-\mu(\overline{\omega}_{t-1})} + \frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}(1-\theta)\right)\mu'(\omega)\mathrm{d}\omega +$$

$$\int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}} \frac{\left(R_{t-1}^B-\xi\right)\left(a_{t-1}(\omega)+\Theta_t(\omega)b_{t-1}(\omega)\right)}{a_{t-1}(\omega)}\mu'(\omega)\mathrm{d}\omega +$$

$$\int_{\overline{\overline{\omega}}_{t-1}}^{1} \frac{R_t(\omega)\left(a_{t-1}(\omega)+b_{t-1}(\omega)\right)-\rho_{t-1}b_{t-1}(\omega)}{a_{t-1}(\omega)}\mu'(\omega)\mathrm{d}\omega, \quad \text{(B.1)}$$

where we use equations (25), (21), and (19) to size the equity returns on each segment of firms.

Simplifying

$$R_t^A = \rho_{t-1}\frac{1-\mu(\overline{\overline{\omega}}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}\mu(\overline{\omega}_{t-1}) + \frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}(1-\theta)\mu(\overline{\omega}_{t-1}) +$$

$$\int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}} \frac{(R_{t-1}^B-\xi)d_{t-1}(\omega)}{a_{t-1}}\mu'(\omega)\mathrm{d}\omega + \int_{\overline{\overline{\omega}}_{t-1}}^{1} \frac{R_t(\omega)(a_{t-1}+b_{t-1})}{a_{t-1}}\mu'(\omega)\mathrm{d}\omega - \int_{\overline{\overline{\omega}}_{t-1}}^{1} \frac{\rho_{t-1}b_{t-1}}{a_{t-1}}\mu'(\omega)\mathrm{d}\omega =$$

$$\rho_{t-1}\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{b_{t-1}}{a_{t-1}} + \left(\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})\right)(1-\theta)\frac{b_{t-1}}{a_{t-1}} + \frac{(R_{t-1}^B-\xi)}{A_{t-1}}D_{t-1} +$$

$$\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{\left(A_{t-1}+\frac{B_{t-1}}{1-\mu(\overline{\omega}_{t-1})}\right)}{A_{t-1}}\int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t(\omega)\mu'(\omega)\mathrm{d}\omega - \rho_{t-1}\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{b_{t-1}}{a_{t-1}} =$$

$$(1-\theta)\frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}\frac{B_{t-1}}{A_{t-1}} + \frac{(R_{t-1}^B-\xi)}{A_{t-1}}D_{t-1} +$$

$$\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{\left(A_{t-1}+\frac{B_{t-1}}{1-\mu(\overline{\omega}_{t-1})}\right)}{A_{t-1}}\int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t(\omega)\frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})}\mathrm{d}\omega =$$

$$(1-\theta)\frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}\frac{B_{t-1}}{A_{t-1}} + \frac{(R_{t-1}^B-\xi)}{A_{t-1}}D_{t-1} + \frac{B_{t-1}^{tot}}{A_{t-1}}R_t, \quad \text{(B.2)}$$

where we use $a_{t-1}(\omega) = a_{t-1}$ and $b_{t-1}(\omega) = b_{t-1}$ for all ω and equations (45),(46), (49), and (54). Moreover, we use that

$$\frac{b_{t-1}}{a_{t-1}} = \frac{\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} = \frac{B_{t-1}}{A_{t-1}}\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} \quad \text{(B.3)}$$

by combining equations (45), (46), and (44).

We can re-write equation (B.2) as follows:

$$R_t^A A_{t-1} = B_{t-1}^{tot}R_t + (R_{t-1}^B-\xi)D_{t-1} + (1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}B_{t-1}. \quad \text{(B.4)}$$

B.2 Beta Distribution: Aggregate Output

Plugging the functional form of the distribution into equation (58) and taking the integral:

$$Y_{t+1} = \frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1}\omega K_t^{\alpha} H_{t+1}^{1-\alpha} \mu'(\omega)\mathrm{d}\omega =$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1}\omega K_t^{\alpha} H_{t+1}^{1-\alpha} \frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)}\mathrm{d}\omega =$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1} K_t^{\alpha} H_{t+1}^{1-\alpha} \frac{\omega^{\eta_1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1)\Gamma(\eta_2)}\Gamma(\eta_1+\eta_2)\mathrm{d}\omega =$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1} K_t^{\alpha} H_{t+1}^{1-\alpha} \frac{\omega^{\eta_1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1+1)\Gamma(\eta_2)}\Gamma(\eta_1+\eta_2+1)\frac{\Gamma(\eta_1+1)}{\Gamma(\eta_1)}\frac{\Gamma(\eta_1+\eta_2)}{\Gamma(\eta_1+\eta_2+1)}\mathrm{d}\omega =$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1} K_t^{\alpha} H_{t+1}^{1-\alpha} \frac{\omega^{\eta_1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1+1)\Gamma(\eta_2)}\Gamma(\eta_1+\eta_2+1)\frac{\eta_1\Gamma(\eta_1)}{\Gamma(\eta_1)}\frac{\Gamma(\eta_1+\eta_2)}{(\eta_1+\eta_2)\Gamma(\eta_1+\eta_2)}\mathrm{d}\omega =$$

$$\frac{\eta_1}{\eta_1+\eta_2}\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} Z_{t+1}K_t^{\alpha}H_{t+1}^{1-\alpha}, \quad \text{(B.5)}$$

where we use the property of the gamma function: $\Gamma(z+1) = z\Gamma(z)$.

B.3 Beta Distribution: Average Return on Capital

Plugging the functional form of the distribution into equation (61) and taking the integral:

$$R_t = \int_{\overline{\overline{\omega}}_t}^{1} R_t(\omega)\frac{\mu'_{\eta_1,\eta_2}(\omega)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}\mathrm{d}\omega =$$

$$\int_{\overline{\overline{\omega}}_t}^{1} \left(\frac{1}{Q_{t-1}}\alpha\ Z_t\left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha}\omega + \frac{(1-\delta)}{Q_{t-1}}Q_t\right)\frac{\mu'_{\eta_1,\eta_2}(\omega)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}\mathrm{d}\omega = \quad \text{(B.6)}$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}\int\limits_{\overline{\overline{\omega}}_t}^{1}\frac{1}{Q_{t-1}}\alpha\ Z_t\left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha}\omega\frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)}\mathrm{d}\omega+$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}\int\limits_{\overline{\overline{\omega}}_t}^{1}\frac{(1-\delta)}{Q_{t-1}}Q_t\frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)}\mathrm{d}\omega=$$

$$\frac{\eta_1}{\eta_1+\eta_2}\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)}\frac{1}{Q_{t-1}}\alpha\ Z_t\left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha}+\frac{(1-\delta)}{Q_{t-1}}Q_t \quad \text{(B.7)}$$

where we use the similar manipulations as before to calculate the integral.

B.4 Beta Distribution: Aggregate Equity Return

Plugging the functional form of the distribution into equation (B.1) and taking the integral:

$$R_t^A=\int\limits_{0}^{\overline{\omega}_{t-1}}\left(\rho_{t-1}\frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}+\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}(1-\theta)\right)\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega+$$

$$\int\limits_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}}\frac{\left(R_{t-1}^B-\xi\right)\left(a_{t-1}(\omega)+\Theta_t(\omega)b_{t-1}(\omega)\right)}{a_{t-1}(\omega)}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega+$$

$$\int\limits_{\overline{\overline{\omega}}_{t-1}}^{1}\frac{R_t(\omega)\left(a_{t-1}(\omega)+b_{t-1}(\omega)\right)-\rho_{t-1}b_{t-1}(\omega)}{a_{t-1}(\omega)}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega= \quad \text{(B.8)}$$

$$\rho_{t-1}\frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})+\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}(1-\theta)\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})+$$

$$\int\limits_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}}\frac{(R_{t-1}^B-\xi)d_{t-1}(\omega)}{a_{t-1}}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega+$$

$$\int\limits_{\overline{\overline{\omega}}_{t-1}}^{1}\frac{R_t(\omega)(a_{t-1}+b_{t-1})}{a_{t-1}}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega-\int\limits_{\overline{\overline{\omega}}_{t-1}}^{1}\frac{\rho_{t-1}b_{t-1}}{a_{t-1}}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega= \quad \text{(B.9)}$$

B.5 Beta Distribution: Diversion Function

Expressing F_t from equation (68):

$$\theta = \frac{1}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \omega^{\psi} F_t \frac{\omega^{\eta_1 - 1}(1-\omega)^{\eta_2 - 1}}{B(\eta_1, \eta_2)} \mathrm{d}\omega =$$

$$\frac{1}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} F_t \frac{\omega^{\eta_1 + \psi - 1}(1-\omega)^{\eta_2 - 1}}{\Gamma(\eta_1)\Gamma(\eta_2)} \Gamma(\eta_1 + \eta_2) \mathrm{d}\omega =$$

$$\frac{1}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} F_t \frac{\omega^{\eta_1 + \psi - 1}(1-\omega)^{\eta_2 - 1}}{\Gamma(\eta_1 + \psi)\Gamma(\eta_2)} \Gamma(\eta_1 + \psi + \eta_2) \frac{\Gamma(\eta_1 + \eta_2)\Gamma(\eta_1 + \psi)}{\Gamma(\eta_1)\Gamma(\eta_1 + \psi + \eta_2)} \mathrm{d}\omega =$$

$$\frac{F_t}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \frac{\Gamma(\eta_1 + \eta_2)\Gamma(\eta_1 + \psi)}{\Gamma(\eta_1)\Gamma(\eta_1 + \psi + \eta_2)} \left(\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega}_t)\right) \quad \text{(B.10)}$$

Therefore,

$$F_t = \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega}_t)} \frac{\Gamma(\eta_1)\Gamma(\eta_1 + \psi + \eta_2)}{\Gamma(\eta_1 + \eta_2)\Gamma(\eta_1 + \psi)} \theta \quad \text{(B.11)}$$

C Equilibrium Conditions

$$\lambda_{ct} = \beta E_t \left\{ \lambda_{ct+1} R^A_{t+1} \right\}, \tag{C.1}$$

$$\beta E_t \left\{ \lambda_{ct+1} \left(R^B_t + FS_t \right) \right\} = \left(C_t - \vartheta \frac{H_t^{1+\nu}}{1+\nu} \right)^{-\sigma}, \tag{C.2}$$

$$W_t = \vartheta H_t^{\nu}, \tag{C.3}$$

$$\lambda_{ct} = \beta E_t \left\{ \lambda_{ct+1} R^B_t \right\}, \tag{C.4}$$

$$Y_t = \frac{\eta_1}{\eta_1 + \eta_2} \frac{1 - \mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})} Z_t K^{\alpha}_{t-1} H_t^{1-\alpha}, \tag{C.5}$$

$$B^{tot}_t = Q_t K_t, \tag{C.6}$$

$$W_t = (1-\alpha) \frac{Y_t}{H_t}, \tag{C.7}$$

$$R_t = \frac{\eta_1}{\eta_1 + \eta_2} \frac{1 - \mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_t)}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \frac{1}{Q_{t-1}} \alpha \; Z_t \left(\frac{H_t}{K_{t-1}} \right)^{1-\alpha} + \frac{(1-\delta)}{Q_{t-1}} Q_t, \tag{C.8}$$

$$I^n_t = \left[1 - \frac{\phi}{2} \left(\frac{I^g_t}{I^g_{t-1}} - 1 \right)^2 \right] I^g_t, \tag{C.9}$$

$$K_t = I^n_t + (1-\delta) K_{t-1}, \tag{C.10}$$

$$\Pi_t = Q_t \left[1 - \frac{\phi}{2} \left(\frac{I^g_t}{I^g_{t-1}} - 1 \right)^2 \right] I^g_t - I^g_t, \tag{C.11}$$

$$T_t = B^G_t - R^B_{t-1} B^G_{t-1}, \tag{C.12}$$

$$B^G_t = B^H_t + D_t, \tag{C.13}$$

$$B^H_t = 0, \tag{C.14}$$

$$\Xi_t = \xi D_{t-1}, \tag{C.15}$$

$$E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(\frac{1}{Q_t} \alpha \; Z_{t+1} \left(\frac{H_{t+1}}{K_t} \right)^{1-\alpha} \overline{\omega}_t + \frac{(1-\delta)}{Q_t} Q_{t+1} \right) \right] = E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R^B_t - \xi \right) \right], \tag{C.16}$$

$$L_t = \mu_{\eta_1,\eta_2}(\overline{\omega}_t) A_t, \tag{C.17}$$

$$B_t = L_t, \tag{C.18}$$

$$D_t = \left(\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t) \right) A_t + \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \theta B_t, \tag{C.19}$$

$$B^{tot}_t = \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) \right) A_t + \frac{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} B_t, \tag{C.20}$$

$$R_t^A A_{t-1} = B_{t-1}^{tot} R_t + (R_{t-1}^B - \xi) D_{t-1} + (1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1}) - \mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} B_{t-1}, \tag{C.21}$$

$$Y_t = C_t + I_t^g, \tag{C.22}$$

$$\left(\rho_t \frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)} + (1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right) A_t = \left(R_t^B - \xi\right)\left(A_t + F_t \overline{\omega}_t^{\psi} \frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right), \tag{C.23}$$

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B - \xi\right)\left(A_t + F_t \overline{\overline{\omega}}_t^{\psi} \frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right)\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\overline{\overline{\omega}}_t + \frac{(1-\delta)}{Q_t}Q_{t+1}\right)\left(A_t + \frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right) - \rho_t \frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right)\right], \tag{C.24}$$

$$0 = E_t\left\{\begin{array}{c} Q_t\left[-\phi\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)\frac{1}{I_{t-1}^g}\right] I_t^g + Q_t\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - 1 \\ +\beta\frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1}\phi\left(\frac{I_{t+1}^g}{I_t^g} - 1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \end{array}\right\}, \tag{C.25}$$

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z, \tag{C.26}$$

$$FS_t = \rho_f FS_{t-1} + \varepsilon_t^f, \tag{C.27}$$

where

$$F_t = \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega}_t)} \frac{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)}{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)}\theta \tag{C.28}$$

and ω follows the Beta distribution with parameters η_1 and η_2, i.e.,

$$\mu'_{\eta_1,\eta_2}(\omega) = \frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)},$$

where $B(\eta_1,\eta_2) = \frac{\Gamma(\eta_1)\Gamma(\eta_2)}{\Gamma(\eta_1+\eta_2)}$ and Γ is the Gamma function.

D Steady-State Conditions

We derived 28 equilibrium conditions for 28 endogenous variables:

$\lambda_{ct}, R_t^A, C_t, H_t, W_t, R_t^B, Y_t, \overline{\omega}_t, \overline{\overline{\omega}}_t, Z_t, FS_t, K_t, B_t^{tot}, Q_t, R_t, I_t^n, I_t^g, \Pi_t, T_t, B_t^G, B_t^H, D_t, \Xi_t, L_t, A_t,$
$B_t, \rho_t,$ and F_t.

Next, we consider a strategy for finding the non-stochastic steady state. The strategy will involve guessing the values of $\overline{\omega}$, $\overline{\overline{\omega}}$, and R and iterating to a fixed point.

Given our guesses, we can find

$$F = \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega})} \frac{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)}{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)}\theta \tag{D.1}$$

Continue by fixing

$$H = H^{ss} = 1. \tag{D.2}$$

We can support any choice of H by choosing ϑ appropriately. For example, by dividing equation (C.3) by equation (C.2) and evaluating the ratio in the steady state, we can find that $\vartheta = \frac{W}{H^{\nu}}$ supports $H = 1$ in the steady state.
From equation (C.4), we can see that in the steady state

$$R^B = \frac{1}{\beta}. \tag{D.3}$$

Similarly, from equation (C.1)

$$R^A = \frac{1}{\beta}. \tag{D.4}$$

From equation (C.26)

$$Z = 1. \tag{D.5}$$

From equation (C.27)

$$FS = 0. \tag{D.6}$$

From equation (C.25)

$$Q = 1. \tag{D.7}$$

From equation (C.8) in the steady state

$$R = \frac{\eta_1}{\eta_1+\eta_2}\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}\alpha\left(\frac{H}{K}\right)^{1-\alpha} + 1-\delta,$$

we can solve for K

$$K = H\left[(R-(1-\delta))\frac{\eta_1+\eta_2}{\alpha\eta_1}\frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}})}\right]^{\frac{1}{\alpha-1}}. \tag{D.8}$$

Combining equation (C.6) and our result (D.7), we get

$$B^{tot} = K. \tag{D.9}$$

From equation (C.5)

$$Y = \frac{\eta_1}{\eta_1+\eta_2}\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}K^{\alpha}H^{1-\alpha} \tag{D.10}$$

From equation (C.7)

$$W = (1-\alpha)\frac{Y}{H}. \tag{D.11}$$

From equation (C.10)

$$I^n = \delta K. \tag{D.12}$$

From equation (C.9)

$$I^g = I^n. \tag{D.13}$$

Therefore, from equation (C.22), we can find

$$C = Y - I^g. \tag{D.14}$$

Therefore, from equation (C.2), we can find

$$\lambda_c = \left(C - \vartheta\frac{H^{1+\nu}}{1+\nu}\right)^{-\sigma}. \tag{D.15}$$

Combining equations (C.17) and (C.18) and plugging the result into equation (C.20),

$$B^{tot} = (1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}))\, A + \frac{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})} \mu_{\eta_1,\eta_2}(\overline{\omega}) A$$

we can find A

$$A = \frac{B^{tot}}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) + \frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})} \mu_{\eta_1,\eta_2}(\overline{\omega})}. \tag{D.16}$$

Therefore, from the combination of equations (C.17) and (C.18), we can find

$$B = \mu_{\eta_1,\eta_2}(\overline{\omega}) A. \tag{D.17}$$

From equation (C.18)

$$L = B. \tag{D.18}$$

From equation (C.19)

$$D = (\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega}))\, A + \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})} \theta B. \tag{D.19}$$

From equation (C.14)

$$B^H = 0. \tag{D.20}$$

Therefore, from equation (C.13)

$$B^G = D. \tag{D.21}$$

From equation (C.12)

$$T = B^G - R^B B^G, \tag{D.22}$$

From equation (C.15)

$$\Xi = \xi D. \tag{D.23}$$

From equation (C.11)

$$\Pi = 0. \tag{D.24}$$

From equation (C.23)

$$\left(\rho\frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}+(1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})-\mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right)A=\left(R^B-\xi\right)\left(A+\frac{F\overline{\omega}^{\psi}B}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right),$$

we can find

$$\rho=\frac{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}\left(\left(R^B-\xi\right)\left(1+\frac{F\overline{\omega}^{\psi}\mu_{\eta_1,\eta_2}(\overline{\omega})}{(1-\mu_{\eta_1,\eta_2}(\overline{\omega}))}\right)-(1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})-\mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right), \quad (D.25)$$

where we use equation (D.17) to substitute for $\frac{B}{A}$.

We have 3 equations to verify our 3 guesses

1. From equation (C.16), verify the guess for $\overline{\omega}$

$$\alpha\left(\frac{H}{K}\right)^{1-\alpha}\overline{\omega}+1-\delta=R^B-\xi,$$

which can be re-written as

$$0=-1+\frac{R^B-\xi-(1-\delta)}{\alpha\left(\frac{H}{K}\right)^{1-\alpha}\overline{\omega}}. \quad (D.26)$$

2. From equation (C.24), verify the guess for $\overline{\overline{\omega}}$

$$\left(R^B-\xi\right)\left(A+F\overline{\overline{\omega}}^{\psi}\frac{B}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right)=\left(\alpha\left(\frac{H}{K}\right)^{1-\alpha}\overline{\overline{\omega}}+1-\delta\right)\left(A+\frac{B}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right)-\rho\frac{B}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})},$$

which can be re-written as

$$0=-1+\left(\alpha\left(\frac{H}{K}\right)^{1-\alpha}\overline{\overline{\omega}}+1-\delta\right)\frac{1}{\rho}\left(\frac{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}{\mu_{\eta_1,\eta_2}(\overline{\omega})}+1\right)-\left(R^B-\xi\right)\frac{1}{\rho}\left(\frac{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}{\mu_{\eta_1,\eta_2}(\overline{\omega})}+F\overline{\overline{\omega}}^{\psi}\right), \quad (D.27)$$

where we use equation (D.17) to substitute for $\frac{A}{B}$

3. From equation (C.21), verify the guess for R

$$R^A A = B^{tot} R + (R^B - \xi) D + (1-\theta) \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})} B,$$

which can be re-written as

$$0 = -1 + \frac{B^{tot} R}{R^A A} + \frac{(R^B - \xi) D}{R^A A} + (1-\theta) \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{(1 - \mu_{\eta_1,\eta_2}(\overline{\omega})) R^A A} B. \tag{D.28}$$

E Data and Model Counterparts

We are interested in the segment for the producing firms. To calculate the 90^{th} percentile for that segment, we need to rescale the density so that it integrates to 1 over the segment. Accordingly, for the 10^{th} percentile, we can solve for ω_{10} from

$$\int_{\overline{\overline{\omega}}}^{\omega_{10}} \frac{\mu'_{\eta_1,\eta_2}(\omega)}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})} \mathrm{d}\omega = 0.10. \tag{E.29}$$

Working on the derivation:

$$\frac{1}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})} \left(\mu_{\eta_1,\eta_2}(\omega_{10}) - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) \right) = 0.10.$$

Hence,

$$\omega_{10} = \mu^{-1}_{\eta_1,\eta_2} \left[0.10 \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) \right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) \right], \tag{E.30}$$

where $\mu^{-1}_{\eta_1,\eta_2}$ is the inverse of the Beta distribution.

Analogously for the 90^{th} percentile, the expression will be

$$\omega_{90} = \mu^{-1}_{\eta_1,\eta_2} \left[0.90 \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) \right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) \right]. \tag{E.31}$$

Therefore, our target ratio is

$$\frac{\omega_{90}}{\omega_{10}} = \frac{\mu^{-1}_{\eta_1,\eta_2} \left[0.90 \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) \right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) \right]}{\mu^{-1}_{\eta_1,\eta_2} \left[0.10 \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) \right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) \right]}. \tag{E.32}$$