A Static Capital Buffer is Hard To Beat*

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Abstract

In a model with endogenous risk-taking, deposit insurance and limited liability may lead banks to make risky loans that are socially inefficient. Capital requirements can prevent excessive risk-taking at the cost of reducing liquidity producing bank deposits. A policy that sets capital requirements just high enough to prevent excessive risk-taking will move capital requirements pro-, counter-, or a-cyclically depending on the shock source. However, such a policy requires full knowledge of all the shocks hitting the economy and is not implementable. Simple rules that respond to cyclical conditions—in line with Basel III guidance—perform poorly, whereas a small static capital buffer can do much better.

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First, do no harm, Hippocrates (5th century BCE)

Mind the cliff, Wile E. Coyote (20th century CE)

1 Introduction

A protracted period of low real returns on safe assets followed in the wake of the Global Financial Crisis and was only interrupted by the COVID inflation surge. The return of low interest rates could tempt financial intermediaries to reach for higher yields by taking excessive (or socially inefficient) risks. The risk-taking behavior that we have in mind is epitomized by the emergence of NINJA loans in the subprime mortgage market leading up to the Great Recession, and by the booming of leveraged loans in its aftermath.

To study these concerns, we develop a dynamic macroeconomic model in which limited liability and deposit insurance provide incentives for a bank to shift from safe assets to risky assets in its portfolio of loans.[1] More specifically, and following Van den Heuvel (2008), our banks can lend to safe firms or risky firms. As in that work, risky firms are exposed to an idiosyncratic shock with negative expected value. A profit maximizing bank could fund a firm with negative expected value only because limited liability shields it from downside risk and because deposit insurance takes away the incentives of depositors to monitor the activities of banks. An important extension in our model is that both safe and risky firms face aggregate shocks, allowing the model to capture aggregate fluctuations. In response to shocks, if capital requirements are not sufficiently high, financing risky loans can temporarily become attractive and lead to banking crises in which some banks fail and deposit insurance bails out depositors.[2]

We use the simulated method of moments to calibrate the model. Our estimation sample runs from the first quarter of 1980 to the fourth quarter of 2024. Over this period, capital requirements did not systematically respond to macroeconomic conditions. We model them as a static buffer over the optimal steady-state rate. We calibrate capital requirements as the average capital-to-asset ratio in the banking sector. We choose the parameters governing various sources of shocks, including the volatility of idiosyncratic shocks that make risky projects attractive, to match the average bank failure rate over the sample and other moments

[1] We do not analyze the optimality of either limited liability or bank deposit insurance; we simply take them as features of the economy. Our model would not be adequate for a discussion of deposit insurance since we exclude the possibility of bank runs.

[2] This is another important extension relative to Van den Heuvel (2008), who only considers excessive risk-taking as an off-equilibrium outcome, ruled out by sufficiently high capital requirements. While that work came on the heels of a prolonged period of stability, the more recent experience has offered painful evidence that banking crises do still occur.

for key macroeconomic indicators, such as the volatility of GDP and investment. We also ensure that periods of excessive risk-taking and elevated bank failures are consistent with the average decline in GDP relative to trend observed for such periods. In our model as in the observed data, periods with de minimis failure rates are punctuated by crisis periods with elevated failure rates.

Banking crises in our model have major consequences for household consumption and business investment. Bank capital requirements can curb the risk-taking incentives that lead to crises, and indeed changes to capital requirements continue to engage the policy and academic communities.[3] In our model, very high capital requirements force a bank to keep enough "skin in the game" to eliminate the excessive risk-taking incentives entirely. But capital requirements also reduce bank deposits, which provide liquidity services to households.

Using this model, our contribution is to compare the performance of different rules—including simple and implementable rules—in the realistic situation of a constellation of shocks bombarding the economy at the same time. The best-performing rule we could devise respecting the constraints of a decentralized equilibrium maximizes the liquidity value of deposits while avoiding costly risk-taking episodes and bank failures. Accordingly, this rule sets capital requirements just high enough for bankers to have sufficient skin in the game to avoid excessive risk-taking and bank failures; we dub this rule "no-failure." Beyond the level that prevents excessive risk-taking, capital requirements would bring no additional benefits and would supplant deposits that are valued by households for their liquidity. Unrealistically, this rule requires full knowledge of the shocks hitting the economy. By contrast, simple implementable rules that adjust capital requirements in response to a few observable macroeconomic indicators cannot always prevent crises and thus do worse despite allowing household to hold higher deposit balances in some periods. In this context, a static buffer turns out to be a sound, simple, and implementable policy.

So, what are the dynamic capital requirements of this best-performing rule? Triggering a risk-taking episode would lower household utility by a discrete amount. Accordingly, a regulator armed with perfect knowledge of the shock and the structure of the economy—faced with aggregate and firm-specific shocks—could increase capital requirements just enough to avoid triggering a risk-taking episode. A less-informed regulator in the real world might be tempted to zip up to the estimate of the cliff's edge. But this well-meaning regulator could face a Wile E. Coyote moment, and may be better advised to exercise caution against banking crises, or do no harm.

[3]Following a pause in rule making during the Covid pandemic, many jurisdictions are moving to the finalization phase of the Basel III agreements. We review the related academic literature later in the introduction.

The no-failure rule may be intuitive even if it is not implementable, but is it optimal in any sense? We start by showing that, for a static version of our model that excludes aggregate shocks, the steady-state capital requirement implied by our rule is the global optimum. The verification of global optimality in the full model is more complex. One heuristic check is whether a flexible rule that can respond to any possible linear combination of model variables is welfare superior. This flexible rule sets capital requirements as a function of lagged capital requirements, all (lagged) state variables of the model, and all shock processes, including their innovations. Choosing the coefficients of this rule to maximize welfare yields a rule that calls for capital requirements that are perfectly correlated with the capital requirements set by our candidate optimal rule. Admittedly, this flexible rule is optimal only in the class of linear rules with constant coefficients.[4] Moreover, by requiring a response to unobservable shocks, this flexible rule is not implementable.

To characterize the properties of our best-performing rule, we begin by showing how it responds to individual macroeconomic shocks. We provide examples in which our best-performing rule would raise capital requirements: (1) during a downturn caused by a productivity shock; (2) during an expansion caused by an investment-specific shock; or (3) during an increase in the volatility of financial market returns that has little effect on the business cycle. So, the best-performing rule in the class of linear rules would not necessarily set capital requirements pro-, counter-, a-cyclically. This is the basic reason why, as we shall see, simple countercyclical rules for setting capital requirements do poorly in our model.

We compare the performance of our best-performing rule with that of simple and implementable rules optimized to maximize welfare conditional on the inclusion of a small number of aggregate indicators. Of particular interest is the Basel III guidance for setting the countercyclical capital buffer (CCyB). According to this guidance, capital requirements should increase during periods of rapid credit expansion (or increases in the credit-to-GDP ratio), and they should be relaxed during a credit contraction.[5] This guidance—which we will call the "Basel rule"—sounds both sensible and implementable. And indeed, some statistical correlations would seem to support it. For example, in line with the statistical evidence that provided the underpinnings for the Basel Rule, the credit-to-GDP ratio (weakly) predicts GDP two years hence.[6] We show that this predictability is both a feature of the observed data and of our model. Nonetheless, grounding the statistical underpinnings for the Basel

[4] Ruling out the possibility that nonlinear rules that allow coefficients to vary when banks fail may enhance welfare is beyond the scope of our paper.

[5] The guidelines can be found in Basel Committee on Banking Supervision (2010).

[6] Borio and Lowe (2002) and Borio and Lowe (2004) provided the empirical underpinnings for the Basel III guidance. Their findings were bolstered by subsequent empirical work in Schularick and Taylor (2012), Jordà et al. (2017), Mian et al. (2017), and Mian et al. (2020).

guidance in a theoretical framework leads to different policy prescriptions. Optimizing this rule results in a vanishingly small coefficient on the credit-to-GDP ratio, making the rule indiscernible from a static buffer.

Actually, none of the simple dynamic rules that we consider improve meaningfully on the performance of a simple static buffer. A planner allowed to optimize the coefficients of the dynamic rules either abstains from introducing much variation in capital requirements or compensates for that possibly misguided variation by elevating the average level of capital. We will see that a slightly elevated static capital requirement (or buffer) largely avoids the Wile E. Coyote moments, and it does almost as well as the optimal linear rule in welfare terms.

There are several strands of literature related to our work. The papers by Begenau (2020), Begenau and Landvoigt (2022), Collard et al. (2017), Martinez-Miera and Suarez (2012), Mishin (2023), and Pancost and Robatto (2023) share the risk-shifting framework in our model. Begenau (2020) and Pancost and Robatto (2023) —arguably the closest analyses to our own—focus on the optimal level of static capital requirements in the steady state. Our focus is on cyclical capital requirements, and their comparison with static buffers. Begenau and Landvoigt (2022) and Mishin (2023) center on the interactions between regulated and unregulated banks. Martinez-Miera and Suarez (2012) develop a model with systemic risk in the form of a binary shock, which simplifies the theoretical derivations and numerical solution. By contrast, our model has a richer stochastic structure, important for our assessment of simple rules, something they do not attempt. Collard et al. (2017) concentrate on interactions of optimal monetary and prudential policies, in a setting that keeps bank failures off the equilibrium path. We abstract from monetary policy, but we allow for business-cycle fluctuations and risk-taking on the equilibrium path.

We also contribute to the body of work that shows how regime shifts can lead to discontinuous outcomes, financial crises, without relying on discontinuous distributions for exogenous shocks. See, for example, Mendoza (2010), He and Krishnamurthy (2012), and Brunnermeier and Sannikov (2014). Our approach relies on endogenous regime shifts leading to banking crises, discontinuous events in which a large set of banks go bankrupt, and deposit insurance intervenes to bail out depositors. Moreover, there is a literature on credit booms and busts, including Boissay et al. (2016) and Bordalo et al. (2018). We differ mainly by not (explicitly) modeling banking panics and by integrating the analysis within a reasonably conventional quantitative macroeconomic framework with a clear role for capital regulation of banks.

Several influential contributions to the literature emphasize risks arising from high leverage and the expansion of bank credit. Davydiuk (2017) and Malherbe (2020) are examples of this. Our work offers a complementary perspective that emphasizes the composition of bank credit, rather than its expansion. Gomes et al. (2023) develop a model that shares our emphasis on risks arising from changes in the composition of bank credit, rather than the expansion of credit. They question the premise that high leverage causes banking crises and that curbing credit growth can prevent crises. In their model, a time-varying likelihood of an exogenous economic crisis causes both higher leverage and the subsequent economic decline, making policies that respond to credit growth ineffective.

Other contributions show how leverage can increase financial fragility and the risk of bank runs. Examples include Angeloni and Faia (2013), Gertler and Kiyotaki (2015), Gertler et al. (2020), and Faria-e-Castro (2021). We make our formal analysis stark by setting aside bank runs, but of course we recognize the possibility of bank runs in reality.

The rest of our paper proceeds as follows. Section 2 describes the model and presents an analytical characterization of aspects of the model's equilibrium. Section 3 discusses the model's calibration and solution method. Section 4 characterizes the benchmark no-failure capital policy and related optimality tests—our second set of important results. Section 5 presents the responses to different shocks and discusses the no-failure policy for capital requirements. As for our final and key results, Section 6 considers the performance of some simple implementable rules. Section 7 concludes.

2 The Model

Our model extends a standard RBC model to include banks that enjoy limited liability and government deposit insurance. These are the main features that allow for excessive, or socially inefficient, risk-taking, and the RBC framework allows for macroeconomic shocks that cause business cycles.

Households value deposits and fund banks with equity. Banks provide deposits to households, enjoy limited liability and fund risky and safe firms subject to capital requirements. All firms face aggregate shocks but those that are risky face additional idiosyncratic risk, ε, with negative mean and positive variance. Accordingly, bank returns on loans to risky firms are, on average, lower than the bank returns on safe loans. However, depending on the state of the business cycle, the returns on safe loans might become depressed, and banks could find it attractive to load up on risk because of the shield of limited liability and deposit insurance. Therefore, such risky loans might be privately optimal, despite being socially inefficient. Higher capital requirements can decrease socially inefficient provision of risky loans at the expense of lower liquidity provision from deposits.

We prove analytically that individual banks allocate either the maximum share of their loan portfolios to risky firms or the minimum share to risky firms depending on the state of the business cycle. We dub the former risky banks and the latter safe banks. Safe banks do not fail, whereas risky banks fail at a rate that also varies with the state of the business cycle.

Households solve the portfolio problem to determine the optimal supply of equity across safe and risky banks. Since equity backs loans and banks cannot make negative loans, we impose additional non-negativity constraints on each type of equity to prevent short selling. The solution to this problem equalizes the expected returns across equity types. The demand for equity is pinned down by the binding capital requirements because debt is cheaper than equity.

The deposit insurance scheme is provided by the government and is financed through lump-sum taxes paid by households. The lump-sum nature of these taxes makes the deposit insurance scheme distortive. Deposit insurance and limited liability act as a subsidy for inefficient risk because banks do not internalize the probability of their default on the cost of borrowing. Households view deposits as perfectly safe as they do not internalize their deposit decisions on the financing of the deposit insurance fund.

Banks are at the heart of our model, but the exposition is smoother if we begin with the less-exciting firms and households. In what follows, small letters denote individual households, banks or firms; capital letters represent aggregate values. Safe firms (defined below) carry a superscript s; risky firms carry a superscript r.

2.1 Non-Financial Firms

Non-financial firms are competitive and earn zero profits. There are goods producing firms and capital producing firms. We begin with the former.

2.1.1 Goods Producing Firms

Firms live for just two periods. A firm born in period t, obtains a bank loan, l_t^f, to buy the capital, k_{t+1}, that it will use for production in period $t+1$; so,

$$l_t^f = Q_t k_{t+1}, \tag{1}$$

where Q_t is the price of capital (or the price of investment).[7] The ex-post return on the loan is $R_{t+1} l_t^f = R_{t+1} Q_t k_{t+1}$, where we shall soon see that R_{t+1} is the rate of return on capital ownership. So, these bank loans might be better described as equity positions.

[7] We call bank contracts with the firms loans, but in truth the intermediaries issue equity contracts to the firms, just like in Gertler and Karadi (2011).

There is a continuum of firms of measure 1. But the firms come in two types: "safe" firms face only aggregate shocks, while "risky" firms face both aggregate shocks and idiosyncratic shocks.

In period $t+1$, a safe firm hires labor, h^s_{t+1}, to produce

$$y^s_{t+1} = A_{t+1}(k^s_{t+1})^{\alpha}(h^s_{t+1})^{1-\alpha}, \tag{2}$$

where A_{t+1} is an aggregate shock to total factor productivity (TFP). When a safe firm takes the loan in period t, it knows that the firm will hire the optimal h^s_{t+1} next period. So, the safe firm chooses $l^{f,s}_t$ and k^s_{t+1} in period t, and then h^s_{t+1} in period $t+1$, to

$$\max_{l^{f,s}_t, k^s_{t+1}} \mathbb{E}_t \left\{ \max_{h^s_{t+1}} \left[y^s_{t+1} + (1-\delta)Q_{t+1}k^s_{t+1} - W_{t+1}h^s_{t+1} - R^s_{t+1}l^{f,s}_t \right] \right\}, \tag{3}$$

where δ is the capital depreciation rate, and W_{t+1} is the real wage rate. This maximization is subject to (1) and (2). The first-order conditions for this maximization problem imply

$$\mathbb{E}_t R^s_{t+1} = \alpha \mathbb{E}_t \left\{ \frac{A_{t+1}}{Q_t} \left(\frac{h^s_{t+1}}{k^s_{t+1}} \right)^{1-\alpha} + (1-\delta)\frac{Q_{t+1}}{Q_t} \right\}, \tag{4}$$

where the first term within the brackets is the rental rate on a unit of capital, and the second term is the capital gain on a non-depreciated unit of capital.

A risky firm employs the technology $y^r_{t+1} = A_{t+1}\left(k^r_{t+1}\right)^{\alpha}\left(h^r_{t+1}\right)^{1-\alpha} + \varepsilon_{t+1}k^r_{t+1}$, where ε_{t+1} is an idiosyncratic shock that follows a Normal distribution G with a negative mean, $-\xi$, and standard deviation τ_t:[8]

$$\begin{aligned} &\text{PDF of } \varepsilon_{t+1}, \quad g(\varepsilon_{t+1}) = \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2} \right), \\ &\text{CDF of } \varepsilon_{t+1}, \quad G(\varepsilon_{t+1}) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\varepsilon_{t+1}+\xi}{\tau_t\sqrt{2}} \right) \right]. \end{aligned} \tag{5}$$

In turn, the standard deviation τ_t, in deviation from its mean of τ, is governed by an autoregressive stochastic process of order 1. This stochastic structure is analogous to that of the risk shocks in Christiano et al. (2014).

The risky firm chooses $l^{f,r}_t$ and k^r_{t+1}, and then h^r_{t+1}, to

$$\max_{l^{f,r}_t, k^r_{t+1}} \mathbb{E}_t \left\{ \max_{h^r_{t+1}} \left[y^r_{t+1} + (1-\delta)Q_{t+1}k^r_{t+1} - W_{t+1}h^r_{t+1} - R^r_{t+1}l^{f,r}_t \right] \right\}, \tag{6}$$

[8] $\exp(x) = e^x$ is the exponential function and $\text{erf}(x) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x} \exp\left(-v^2\right) dv = \frac{2}{\sqrt{\pi}}\int_0^x \exp\left(-v^2\right) dv$.

subject to the analogous constraints. The first-order conditions for this maximization, the zero-profit condition for firms, and equation (8) below, imply

$$\mathbb{E}_t R^r_{t+1} = \mathbb{E}_t R^s_{t+1} - \frac{\xi}{Q_t}. \tag{7}$$

So, the idiosyncratic shock lowers the expected value and increases the variance of the return on a loan to a risky firm. Risky loans are socially inefficient, or in our language, excessively risky.

Finally, note that the marginal product of labor for safe and risky firms is given by $(1-\alpha)A_{t+1}(k^i_{t+1}/h^i_{t+1})^\alpha$ where i denotes the type of firm ($i \in \{s, r\}$). Labor is mobile across firms, and both types of firms face the same real wage rate. So, the first-order conditions for labor in period $t+1$ imply the equalization of capital-labor ratios across sectors.

$$k^r_{t+1}/h^r_{t+1} = k^s_{t+1}/h^s_{t+1}. \tag{8}$$

Appendix A.3.3 provides details on the aggregation across firms; we show that there is a representative safe firm that produces

$$Y^s_{t+1} = A_{t+1}(K^s_{t+1})^\alpha (H^s_{t+1})^{1-\alpha}, \tag{9}$$

and also a representative risky firm that produces

$$Y^r_{t+1} = A_{t+1}\left(K^r_{t+1}\right)^\alpha \left(H^r_{t+1}\right)^{1-\alpha} - \xi K^r_{t+1}, \tag{10}$$

where capital letters represent aggregate values.

2.1.2 Capital Producing Firms

At the end of period t, goods producing firms sell their capital to competitive capital producing firms. Letting I^g_t denote gross investment, the evolution of capital follows

$$I_t = \eta_t \left[1 - \frac{\phi}{2}\left(\frac{I^g_t}{I^g_{t-1}} - 1\right)^2\right] I^g_t, \tag{11}$$

where η_t is a shock to investment-specific technology (ISP), and ϕ is a measure of the severity of investment adjustment costs.[9] The aggregate capital stock evolves according to

[9]We include investment adjustment costs, and later habits in consumption, to make our model fit the data better. But they are not an integral part of the logic behind capital requirements.

$$K_{t+1}^{s}+K_{t+1}^{r}=I_{t}+(1-\delta)\left(K_{t}^{s}+K_{t}^{r}\right). \tag{12}$$

The capital producing firms are owned by households, and solve the problem

$$\max_{I_{t+i}^{g}} \mathbb{E}_{t}\sum_{i=0}^{\infty}\psi_{t,t+i}\left\{Q_{t+i}\eta_{t+i}\left[1-\frac{\phi}{2}\left(\frac{I_{t+i}^{g}}{I_{t+i-1}^{g}}-1\right)^{2}\right]I_{t+i}^{g}-I_{t+i}^{g}\right\}, \tag{13}$$

where $\psi_{t,t+i}=\beta\frac{\lambda_{ct+i}}{\lambda_{ct}}$ is the stochastic discount factor of the households, which are described next.

2.2 Households

The economy is populated by a continuum of identical households, each comprising two types of agents: workers and bankers. Workers supply labor to firms, allocate deposits across financial intermediaries and transfer all returns to their household. Bankers manage these intermediaries and likewise return all earnings to the household. All income—whether from saving or banking—is fully pooled within each household, preserving the representative agent structure.[10]

The representative household's problem is

$$\max_{C_{t},D_{t},E_{t}^{s},E_{t}^{r}} \mathbb{E}_{0}\sum_{t=0}^{\infty}\beta^{t}\left[\frac{\left(C_{t}-\kappa C_{t-1}\right)^{1-\varsigma_{c}}-1}{1-\varsigma_{c}}+\varsigma_{0}\frac{D_{t}^{1-\varsigma_{d}}-1}{1-\varsigma_{d}}\right], \tag{14}$$

subject to

$$\begin{aligned} C_{t}+D_{t}+E_{t}^{s}+E_{t}^{r} &= W_{t}+R_{t-1}^{d}D_{t-1}+R_{t}^{e,s}E_{t-1}^{s}+R_{t}^{e,r}E_{t-1}^{r}-T_{t}, \\ E_{t}^{s} &\geq 0, \\ E_{t}^{r} &\geq 0. \end{aligned} \tag{15}$$

Households value consumption, C_t, and value the liquidity services of bank deposits, D_t; β is the discount factor; $0<\kappa<1$ is the habit persistence parameter, $\varsigma_c>0$ captures the intertemporal elasticity of substitution, $\varsigma_0>0$ is the utility weight on deposits, and $\varsigma_d>0$ is the inverse elasticity of household demand for deposits with respect to changes in the interest rate. We put deposits in the utility function in lieu of modeling a particular transaction technology. For simplicity, we assume that households supply labor inelastically,

[10] A detailed exposition of the problems faced by workers and bankers, along with the aggregation into the representative agent framework, is provided in Appendix A.1.

and we have normalized the supply of labor to be one.[11] Household assets include deposits, D_t, which pay a gross real rate R_t^d, and two types of bank equity: E_t^s is equity in a "safe" bank, which lends predominantly to a safe firm and pays $R_{t+1}^{e,s}$ next period; E_t^r is equity in a "risky" bank, which lends predominantly to a risky firm and pays $R_{t+1}^{e,r}$. The returns on equity are of course not known when the household invests. By contrast, the return on deposits is known, and deposits are protected by deposit insurance; deposits are the safe asset in our model. Finally, households pay lump sum taxes, T_t, to fund the government's deposit insurance program.

The household's first-order conditions include:

$$C: \quad (C_t - \kappa C_{t-1})^{-\varsigma_c} - \beta\kappa \mathbb{E}_t (C_{t+1} - \kappa C_t)^{-\varsigma_c} - \lambda_{ct} = 0, \tag{16}$$

$$D: \quad \varsigma_0 D_t^{-\varsigma_d} - \lambda_{ct} + \beta \mathbb{E}_t \{\lambda_{ct+1}\} R_t^d = 0, \tag{17}$$

$$E^s: \quad -\lambda_{ct} + \beta \mathbb{E}_t \{\lambda_{ct+1} R_{t+1}^{e,s}\} + \zeta_t^s = 0, \tag{18}$$

$$E^r: \quad -\lambda_{ct} + \beta \mathbb{E}_t \{\lambda_{ct+1} R_{t+1}^{e,r}\} + \zeta_t^r = 0, \tag{19}$$

where λ_{ct}, ζ_t^s and ζ_t^r are the Lagrangian multipliers for the budget constraint and the two non-negativity constraints. There are also complementary slackness conditions which can be described by:

$$\zeta_t^s E_t^s = 0, \tag{20}$$

$$\zeta_t^r E_t^r = 0. \tag{21}$$

If households did not value deposits for their liquidity services ($\varsigma_0 = 0$), Equation (17) would be the standard RBC Euler condition, and R_t^d would be the standard CAPM rate. But households do value deposits in our model, and R_t^d is below the CAPM rate. Equity is not a safe asset and does not provide liquidity services. So, deposits will be the cheaper source of funding for banks. This fact will play an important role in what follows.

2.3 Banks

Banks are central to our model. First, we set the stage by describing their incentives to take excessive risk. Then, we discuss the banking sector in some detail.

[11] While the total supply of labor is fixed, its distribution across safe and risky firms is market determined.

2.3.1 Incentives to Take Excessive Risk and Capital Requirements

We saw from the section on firms that $\mathbb{E}_t R^r_{t+1} < \mathbb{E}_t R^s_{t+1}$. So, why would a profit-maximizing bank ever invest in a risky firm? Limited liability and government deposit insurance are the culprits here. Limited liability shields the bank from downside risk. Moreover, deposit insurance actually subsidizes risk-taking; it makes bank deposits the safe asset, lowering the cost of issuing deposits, and allowing the bank to expand its portfolio of safe or risky loans. In what follows, we will see that if the expected return on investment in a safe firm falls, due say to a negative TFP shock, the bank may be tempted to take a flier on the risky firm.

As we will see, capital requirements are a potential remedy for excessive risk-taking. In what follows, we will consider a requirement that says equity finance cannot fall below a fraction γ_t of the bank's loans. A high γ_t requires the bank and its equity holders to keep more skin in the game, and it shrinks the bank's portfolio since equity finance is more expensive than deposit finance.

2.3.2 The Banking Sector

A measure one continuum of perfectly competitive banks are born each period, and they live for two periods. In the first period, a bank issues equity, e_t, and deposits, d_t, to households, and uses the proceeds to make loans, l_t, to firms; in the second period, the bank receives the return on its investments and liquidates its assets and liabilities.

More specifically, in period t, the bank creates a loan portfolio by directing a fraction σ_t of its loans to a risky firm; the remainder of its loans go to a safe firm.[12] Since $R^r_{t+1} = R^s_{t+1} + \frac{\varepsilon_{t+1}}{Q_t}$, the ex-post return on the portfolio will be $R^s_{t+1} + \sigma_t \frac{\varepsilon_{t+1}}{Q_t}$.

The bank's net worth in period $t+1$ consists of its earnings on the loan portfolio net of the interest payments on its deposits:

$$nw_{t+1} \equiv \left(R^s_{t+1} + \sigma_t \frac{\varepsilon_{t+1}}{Q_t} \right) l_t - R^d_t d_t. \tag{22}$$

If nw_{t+1} is positive, the bank pays its depositors and distributes the rest to its equity holders. If it is negative, the bank declares bankruptcy; its depositors are protected by deposit insurance, but its equity holders get nothing.

The bank's objective is to maximize the expected return of its equity holders, whose stochastic discount factor is $\psi_{t,t+i}$. Let ε^*_{t+1} be the realization of the idiosyncratic shock

[12] Our assumption that a bank only deals with one safe and one risky firm comes at no loss of generality because all the safe firms are identical, and diversification among the risky firms does not take full advantage of the bank's limited liability. See Collard et al. (2017) for a more formal exposition of this result.

below which the bank's net worth is negative; that is, $\left(R^s_{t+1} + \sigma_t \frac{\varepsilon^*_{t+1}}{Q_t}\right) l_t - R^d_t d_t = 0$. Since the distributions of aggregate and idiosyncratic shocks are independent of each other, we can nest expectations with respect to the idiosyncratic shock within the expectation of the aggregate and idiosyncratic shocks, and the bank's maximization problem can be written as:

$$\max_{l_t, d_t, e_t, \sigma_t} \mathbb{E}_t \left\{ \psi_{t.t+1} \left[\int_{\varepsilon^*_{t+1}}^{\infty} nw_{t+1} \, \mathrm{d}G(\varepsilon_{t+1}) \right] \right\} - e_t, \tag{23}$$

subject to

$$\begin{aligned} l_t &= e_t + d_t, \\ e_t &\geq \gamma_t l_t, \\ l_t &\geq 0, \\ \underline{\sigma} &\leq \sigma_t \leq \overline{\sigma}, \end{aligned} \tag{24}$$

where e_t is equity issued to households. The first constraint is the bank's balance sheet, and the second is the bank's capital requirement. The third constraint rules out short selling; its role will be discussed in Section 3.3. The fourth imposes limits on the fraction of a bank's portfolio that can go to safe or risky loans.[13]

The bank's first-order conditions can be found in Appendix A.2.1. In the next section, we discuss the bank's basic tradeoff when it decides how risky to make its portfolio of loans.

2.3.3 The Bank's Dividends and Its Choice of Risk

In Appendix A.2.5, we show that $\mathit{\Omega}(\sigma_t;\, l_t,\, d_t,\, e_t)$ can be expressed as a linear function of loans, i.e.,

$$\mathit{\Omega}(\sigma_t;\, l_t,\, d_t,\, e_t) = \left(\mathbb{E}_t \left[\psi_{t,t+1} \left(\omega_1 + \omega_2\right)\right] - \gamma_t\right) l_t, \tag{25}$$

where

$$\omega_1 \equiv \left(R^s_{t+1} - R^d_t \left(1 - \gamma_t\right) - \frac{\xi \sigma_t}{Q_t} \right) \left(1 - G(\varepsilon^*_{t+1})\right), \tag{26}$$

$$\omega_2 \equiv \left(\frac{\sigma_t}{Q_t}\right) \frac{\tau_t}{\sqrt{2\pi}} \exp\left(- \left(\frac{\varepsilon^*_{t+1} + \xi}{\tau_t \sqrt{2}} \right)^2 \right), \tag{27}$$

[13]These limits on σ_t are necessary for the numerical methods that follow. In the model calibration, $\overline{\sigma}$ is set equal to 0.99 and $\underline{\sigma}$ is set equal to 0.01; so, banks can get very close to totally safe or totally risky portfolios if they so choose.

and where $1 - G(\varepsilon^*_{t+1})$ is the probability that the bank will not default.

The first component, ω_1, is the return on a loan portfolio with a fraction σ_t going to a risky firm; $-\xi$ is the (negative) expected value of the idiosyncratic shock. The second component, ω_2, is a bonus attributable to the bank's limited liability; the higher is the standard deviation of the idiosyncratic shock, τ_t, the higher is the upside potential for a risky loan, while the downside risk is protected by limited liability.

Increasing σ_t makes the portfolio more risky. More risk decreases the ex-post return on the bank's portfolio, but it increases the bonus from limited liability. This is the tradeoff that a bank faces.

2.4 The Government

The government provides deposit insurance, and collects taxes to pay for it. Given the Ricardian nature of the model, a lump sum tax, T_t, can balance the budget each period without distorting private decision making. In Appendix A.4, we show the tax necessary to support the insurance scheme is

$$T_t = \tfrac{\sigma_{t-1} L_{t-1}}{Q_{t-1}} \tfrac{\tau_t}{\sqrt{2\pi}} \exp\left(- \left(\tfrac{\left(R^d_{t-1}(1-\gamma_{t-1}) - R^s_t\right) Q_{t-1} + \xi \sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_t} \right)^2 \right) - \qquad (28)$$
$$\tfrac{1}{2} L_{t-1} \left(R^s_t - \tfrac{\sigma_{t-1}\xi}{Q_{t-1}} - R^d_{t-1}\left(1-\gamma_{t-1}\right) \right) \left[1 + \operatorname{erf}\left(\tfrac{\left(R^d_{t-1}(1-\gamma_{t-1}) - R^s_t\right) Q_{t-1} + \xi \sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_t} \right) \right],$$

where L_t is the aggregate amount of loans provided by the banking sector. As might be expected, more risk-taking (a higher σ_{t-1}) and/or a higher standard deviation (τ_t) of the idiosyncratic shock increases the taxes required to protect deposits.

2.5 Analytical Characterization of the Equilibrium

We are able to derive some analytical results that enhance our understanding of the model's equilibrium, and how to calculate it. More generally, we will require numerical methods.

2.5.1 Two Propositions and a Corollary

As discussed in the section on households, deposits are a cheaper source of bank funding than equity. So, a bank will fund as much of its loans by issuing deposits as is allowed by the capital requirements. We formalize this argument and prove the following proposition in Appendix A.2.2.

Proposition 1. *In equilibrium, capital requirements always bind; that is,* $e_t = \gamma_t l_t$.

Based on the next proposition, and its corollary, we need only consider two values of the bank's portfolio risk parameter, σ_t, when we derive the model's equilibrium. The proposition is established in Appendix A.2.6.

Proposition 2. *The expected dividend function of banks,* $\Omega(\sigma_t;\ l_t,\ d_t,\ e_t)$*, is convex in* σ_t*. This result holds for arbitrary (and not necessarily continuous) distributions of the idiosyncratic shock.*

Corollary. *There are no equilibria with* $\underline{\sigma} < \sigma_t < \overline{\sigma}$*.*

The intuition for this proposition and its corollary is as follows: If σ_t is high enough, the bank will be bankrupt for low values of ε_t anyway, so it might as well take on as much risk as possible to maximize the portfolio's upside potential for high values of ε_t. If σ_t is low enough, the bank will not be bankrupt even for low values of ε_t, and the value of limited liability is negated; the bank might as well take on the minimum risk to raise the expected value of its portfolio.

Note also that a risky bank seeks to maximize its exposure to the idiosyncratic shock ε_t. Limited liability incentivizes banks to "fail big." So, a risky bank would not want to diversify its loan portfolio by lending to more than one risky firm.[14] At any one time, depending on the state of the economy and the realization of aggregate shocks, only one type of bank may exist, either the risky bank or the safe bank. However, we have not been able to rule out analytically that risky and safe banks may coexist. Accordingly, we allow for this possibility in the numerical solution of the model. Nevertheless, in our model simulations, we have not found any such case.

2.5.2 Equilibrium and Aggregation

We consider a competitive equilibrium in which each bank takes aggregate prices as given. Appendix B lists all the equilibrium conditions of our model. In this section, we only present the equilibrium conditions that are not already included in the preceding sections. We let μ_t denote the fraction of banks with risky portfolios (banks that choose $\sigma_t = \overline{\sigma}$) at date t; the remaining fraction $1 - \mu_t$ are safe banks ($\sigma_t = \underline{\sigma}$).

[14] In reality, bank regulators would not allow a bank to lend to a single firm. But our result really says that risky banks seek exposure to a single idiosyncratic shock ε_t. To circumvent regulation, for example, a bank may hold a seemingly diversified portfolio of MBS with all the loans exposed to the risk of a decrease in house prices. These incentives seem relevant for the literature on securitization surveyed by Gorton and Metrick (2013).

The fraction μ_t is endogenously determined by equity positions of households: we have $\mu_t = \frac{E_t^r}{E_t^r + E_t^s}$. At any point in time, the economy may be in a safe equilibrium (with $\mu_t = 0$), a risky equilibrium (with $\mu_t = 1$), or a mixed equilibrium (with $0 < \mu_t < 1$).

Each bank within a group (safe or risky) is alike and solves the same maximization problem in which it chooses l_t^i, d_t^i, e_t^i according to its type $i \in \{s, r\}$. The aggregate loans to the (representative) safe firm come from two sources: 1) from all safe banks (of measure $1-\mu_t$) that allocate $1 - \underline{\sigma}$ share of their loan portfolio to safe projects and 2) from all risky banks (of measure μ_t) that allocate $1 - \overline{\sigma}$ share of their loan portfolio to safe projects. Therefore, the equilibrium conditions linking our bank-level and firm-level variables representing loans are

$$Q_t K_{t+1}^s = (1 - \underline{\sigma})(1 - \mu_t) l_t^s + (1 - \overline{\sigma}) \mu_t l_t^r. \tag{29}$$

Similarly,

$$Q_t K_{t+1}^r = \underline{\sigma}(1 - \mu_t) l_t^s + \overline{\sigma} \mu_t l_t^r. \tag{30}$$

The aggregate bank loans are linked to the individual bank loans by: $L_t^r = \mu_t l_t^r$ and $L_t^s = (1 - \mu_t) l_t^s$. Therefore, we can describe the latter two equations by using aggregate loans

$$Q_t K_{t+1}^s = (1 - \underline{\sigma}) L_t^s + (1 - \overline{\sigma}) L_t^r, \tag{31}$$

$$Q_t K_{t+1}^r = \underline{\sigma} L_t^s + \overline{\sigma} L_t^r. \tag{32}$$

The equity positions taken by households, in turn, determine the equity positions of individual banks: $E_t^r = \mu_t e_t^r$ and $E_t^s = (1 - \mu_t) e_t^s$. The returns on the equity positions taken by households at date t are linked to the dividends paid by banks at date $t + 1$. We have:

$$\mathbb{E}_t^r R_{t+1}^{e,r} = (\omega_1^r + \omega_2^r) L_t^r, \tag{33}$$

$$\mathbb{E}_t^s R_{t+1}^{e,s} = (\omega_1^s + \omega_2^s) L_t^s, \tag{34}$$

where we use the fact that $\max \left[nw_{t+1}^r, 0 \right]$ is linear in loans; ω_1 and ω_2 were defined in equations (26) and (27). Deposits held by households are issued by (safe and risky) banks: $D_t = D_t^s + D_t^r$ where $D_t^s = L_t^s - E_t^s$ and $D_t^r = L_t^r - E_t^r$.

The equilibrium conditions linking our aggregate and individual firm-specific variables are straightforward, but cumbersome in terms of notation. We state the conditions in Appendix B. The market-clearing conditions for labor, capital, and goods are

$$H_t^s + H_t^r = 1, \tag{35}$$

$$K_t^s + K_t^r = K_t, \tag{36}$$

and

$$Y_t^s + Y_t^r = C_t + I_t^g. \tag{37}$$

3 Simulated Method of Moments, Calibration, and Model Solution

We choose the model parameters with a mix of estimation and calibration. We use the simulated method of moments (SMM) for parameters specific to our model. We pin down some other parameters to hit steady-state targets based on long-run averages. And finally, we calibrate a few parameters based on choices that are common in the literature.

We use the simulated method of moments (SMM) to size the shock processes for total factor productivity, A_t, investment-specific technology, η_t, and for shocks to the volatility of idiosyncratic technology for risky projects, τ_t. We allow each shock to follow an autoregressive process of order 1. We want to size the persistence parameters and the standard deviations of the innovations. We also want to size the investment adjustment cost parameter, ϕ; the habits parameter, κ; capital requirements, γ, modeled as a static buffer; the average standard deviation of the risky firm's idiosyncratic technology shock, τ; and the average penalty for financing risky projects, ξ.

Turning to the implementation of the SMM estimation, the quadratic objective function for the SMM estimation includes the variances, correlations, and autocorrelations for real GDP, real investment, and the relative price of investment.[15] These moments are computed after bandpass-filtering the observed and simulated data (selecting standard business cycle frequencies). The moments are weighted using the SMM optimal weighting matrix with model counterparts computed from a simulated sample spanning 5000 observations. Our data sample runs from the first quarter of 1980 to the fourth quarter of 2024.

The SMM objective function includes two additional quadratic terms that allow us to capture realistic bank failure rates and related declines in economic activity. The first term captures the distance between the average bank failure rate over the observed sample—measured as the assets of failed banks relative to total banking-sector assets and amounting to 1.52 percent on an annualized basis.[16] The second term captures the distance in the

[15] The macroeconomic indicators in our sample are from the National Income and Products Account of the U.S. Bureau of Economic Analysis. We use chain-type indexes. The relative price of investment is the ratio of the price index for gross private domestic investment to the price index for personal consumption expenditures excluding food and energy.

[16] Assets for failed banks are from the FDIC's Bank Failure and Assistance Data. The total assets of the U.S. banking sector are from the Federal Reserve's H.8 Release, Assets and Liabilities of Commercial Banks in the United States.

average decline in economic activity in periods of elevated bank failures in the observed and simulated data. In the observed data, we consider a threshold for failure rates of 1 percent. Selecting only quarters in which the bank failure rate exceeded this threshold in the estimation period, we compute an average real GDP gap (again, using a bandpass filter and selecting standard business cycle frequencies) of -0.33 percent. The weights on these two quadratic terms in the SMM objective function are chosen to be large enough to ensure that the model will match exactly the observed data moments that enter these terms.[17]

We calibrate some parameters to hit steady state targets consistent with averages of observations for key variables over the same estimation sample period as for the SMM procedure. We size γ at 0.078, a value chosen to match the average ratio of equity to assets in the U.S. banking system over the sample period of 7.8 percent.[18] Over this period, capital requirements did not vary systematically with changes in macroeconomic indicators. We capture this feature of capital regulation by simulating data using a simple static buffer for capital requirements. Although all SMM parameters are chosen jointly, effectively, one could interpret the SMM procedure as identifying a buffer above the optimal steady-state capital requirement that is consistent with the observed average bank failure rate given all other parameters.

We set the discount rate, β, at 0.9968, a value consistent with an annualized real risk-free rate of 1.28 percent. This value matches the average interest rate on a 3-month Treasury bill minus the inflation rate over the estimation period.[19]

Finally, the parameter ς_0 enters the household's utility of deposits and influences the tradeoff faced by a planner between the utility of deposits and the disutility of excessive risk-taking. The parameter ς_0 measures the importance of the utility of deposits relative to the utility of consumption. We choose the value of ς_0 of 0.0298, which allows us to match the average spread between the interest rate on a 3-month Treasury bill and the interest rate on deposits rates. For the same period as for the SMM estimation sample, we calculate this average spread to be 0.55% (annualized).[20]

Before discussing the parameters estimated through the SMM procedure, we need to

[17]For the SMM objective function, we scale the quadratic terms for the average bank failure rate and the average GDP gap over periods of elevated bank default rates by sum of the weights of all the other terms entering the objective function.

[18]Equity and asset positions for the U.S. banking sector are from the Federal Reserve's H.8 Release.

[19]Interest rate data are for the 3-month Treasury bill on a discount basis from the secondary market as reported in the Federal Reserve Board's H.15 Release. The inflation data are the quarterly log change in the chain-type price index for personal consumption expenditures excluding food and energy from NIPA Table 2.3.4 of the U.S. Bureau of Economic Analysis.

[20]The 3-month Treasury bill is, again, from the Federal Reserve Board's H.15 Release. The deposit rate is computed as the deposit expense divided by deposits (for domestic and foreign offices) using Call Report data for all FDIC insured banks.

briefly cover how we pinned down the rest of the parameters. Our calibrated parameters are reported at the top of Table 1. These are parameters for which previous papers offer guidance: the capital share $\alpha = 0.3$, the depreciation rate $\delta = 0.025$, the intertemporal elasticity of substitution ϱ_c, and the interest rate elasticity of supply for deposits, ς_d, both set to 1.1 to approximate the log case.

3.1 Simulated Method of Moments, Results

There are no surprises for the parameters from the SMM procedure. They are reported in the bottom part of Table 1. As commonly found in the literature, for instance, the standard deviation of TFP shock is close to 1 percent, and its autoregressive coefficient, at 0.96 indicates a high degree of persistence. There is a high degree of consumption habits, with the parameter κ pinned down at 0.76, whereas we estimate relatively modest investment adjustment costs, with the parameter ϕ at 0.17.

As shown in Table 2, these parameter choices allow the targeted model moments to match closely their counterpart data moments. The model moments reported in the table are computed from a simulated sample of 5000 observations (consistent with the SMM procedure). For each targeted moment, the table also shows the 5^{th} and 95^{th} percentiles from 1000 simulated samples of the same length as the observed data. All but one of the data moments fall between the 5^{th} and 95^{th} percentiles of the simulated moments—the exception is the variance of the price of investment, for which the data moment falls just a bit shy of the 5^{th} percentile.

Armed with the sizes of the shock processes, we can show their relative importance in accounting for the variation of the economic indicators targeted in the SMM procedure and other key variables. As Table 3 shows, the TFP shock is the key driver of the variation in GDP, and investment, whereas the ISP shock accounts for a large share of the variation in the price of investment. As we will show in Section 5.3, the volatility shock plays an important role in plunging the economy into episodes of excessive risk-taking, and optimal capital requirements are therefore quite sensitive to this shock. However, as these episodes of excessive risk-taking are relatively infrequent, the average impact of this shock on aggregate macro variables remains relatively modest, including the impact on the credit-to-GDP ratio. These considerations already point to the possibility that aggregate indicators could provide little information that is relevant for setting capital requirements.

3.2 Untargeted Moments

Given our interest in assessing simple rules for capital requirements that respond to the ratio of credit to GDP, we find it important to assess how well the model can capture key data relationships that underpin the recommendation to consider this type of rule in the Basel III Accords. As shown in Figure 1, there is a predictive relationship between the ratio of non-financial credit to GDP and real GDP two years ahead using data for the United States.[21] The circles in the figure denote observations from the first quarter of 1980 through the fourth quarter of 2024—the same period as for our calibration sample—and point to a tenuous negative correlation, about -0.1. The regression line in the figure confirms that periods in which the credit-to-GDP ratio is above trend systematically presage periods in which GDP will be below trend two years in the future.[22]

To assess whether or not our model is consistent with the negative correlation from U.S. data, we draw from the model 1,000 simulated samples of the same length as the observed data. For each sample, we recompute the regression line shown in Figure 1 and the correlation between the detrended ratio of credit to GDP and GDP two years ahead, which allows us to size a 90 percent confidence interval.[23] Both are included in their respective confidence intervals. Specifically, the interval for the correlation runs approximately from -0.20 to 0.17, thus squarely including the -0.08 correlation based on observed data. We conclude that, even though this is not a moment directly targeted in our calibration based on the simulated method of moments, the model is consistent with the mild predictive relationship between credit and GDP that can be evinced from U.S. data.[24]

In our model, excessive risk-taking leads to negative equity and defaults. Even if the ratio of negative equity to assets at default is not a targeted moment, we can check whether it is consistent with its data counterpart. Using the list of failed FDIC insured banks available from the FDIC, we can compute the equity-to-asset ratio for failed banks within our estimation sample based on the last Call Report filed. We consider a weighted average of this ratio across time.[25] The match between data and model is remarkably good as in

[21] To construct the credit-to-GDP ratio, the credit measure is for private non-financial credit, sourced from the Bank for International Settlements; for GDP we use the nominal volume from National Income and Product Accounts of the U.S. Bureau of Economic Analysis.

[22] We detrended the credit-to-GDP ratio with a Hodrick-Prescott filter using a coefficient of 400,000, implying a trend that is almost linear in line with the prescriptions of the Basel III guidance. This trend is consistent with the recommendations in Borio and Lowe (2002) and Basel III guidance, Basel Committee on Banking Supervision (2010).

[23] We take $L_t^s + L_t^r$ from our model as the measure of credit, and $Y_t^s + Y_t^r$ as the measure of GDP.

[24] Regression results analogous to the ones presented here obtain for various sensitivity exercises, including: a longer sample starting in 1947, using detrended non-financial business credit without dividing it by GDP, and lengthening or shortening the lead length for detrended real GDP.

[25] We deflate nominal assets of failed banks using the consumer price index excluding food and energy. We

both cases the average equity at default is -1.9 percent of assets.[26]

Finally, we consider whether the model predictions on the frequency of periods with elevated failure rates, which we dub "crisis periods" is in line with the observed data. Figure 2 plots the U.S. failure rate weighted by bank assets. We use these data series to find the average failure rate targeted in our SMM exercise. As the figure illustrates, periods with de minimis failure rates are punctuated by periods with elevated failure rates. Using a 1 percent threshold to separate out periods with elevated failure rates—the same threshold as for the conditional output gap in the SMM objective function—the frequency of quarters in which the failure rate is elevated is 7.2 percent. On the model side, the 90-percent confidence interval for periods with excessive risk-taking runs from 2.8 to 14.5 percent, with the average at 8.1 percent. We conclude that the model is a remarkably good match to the data in this dimension despite not being directly targeted in the calibration.

3.3 Model Solution

Occasionally binding non-negativity constraints on bank loans complicate the solution of our model. In our SMM exercise, we need to account for shifts into periods of excessive risk-taking—regime shifts that are jointly determined by the state variables of the model and the realization of exogenous shocks. To address these complications, we rely on the OccBin toolkit developed by Guerrieri and Iacoviello (2015); they also provide an extensive discussion of the accuracy of their solution. In brief, the algorithm reduces the solution of models with occasionally binding constraints to a sequence of indicators denoting whether these constraints are binding or not. Starting from initial conditions and a path for the exogenous shocks, this sequence of indicators is solved for with the same approach as the shooting algorithm of Fair and Taylor (1983). This algorithm can be applied to models with a large number of state variables such as ours.

The Lagrange multipliers χ^i_{2t} on the loan constraints $l^i_t > 0$, where $i \in \{s, r\}$, govern the transition between different regimes, demarcated as follows:

1. Safe regime: $\chi^s_{2t} = 0$, $l^s_t > 0$, $\chi^r_{2t} > 0$, and $l^r_t = 0$,
2. Risky regime: $\chi^s_{2t} > 0$, $l^s_t = 0$, $\chi^r_{2t} = 0$, and $l^r_t > 0$,
3. Mixed regime: $\chi^s_{2t} = 0$, $l^s_t > 0$, $\chi^r_{2t} = 0$, and $l^r_t > 0$.

construct weights by taking the ratio of deflated nominal assets for each banks to the sum of deflated assets for all failed banks.

[26] Bennett and Unal (2015) and Elenev et al. (2021) also discuss the ratio of equity to assets for failed banks.

So, why did we complicate matters by imposing non-negativity constraints on loans? We needed to rule out the short selling of assets (or negative loans). To see why, suppose banks are in the safe regime; in this case, risky loans are overpriced compared to safe loans (because expected returns on risky loans are relatively lower in the safe regime); absent short-selling restrictions, each bank would want to short risky loans. Similar reasoning applies to the risky regime, in which the banks in our model would short safe loans. The last possible scenario is when safe and risky loans are equally priced, so the expected returns on safe and risky loans are the same, resulting in a mixed regime in which $0 < \mu_t < 1$ (as described in Section 2.5.2).

3.3.1 What Triggers a Shift to the Risky Regime?

The answer to this question is rather complex because the banker's maximization problem has so many moving parts. We give a detailed answer in Appendix C; here we offer a simpler explanation that focuses on the main forces at work.

Consider the expected dividends for safe and risky firms, $\Omega_t^s \equiv \mathit{\Omega}(\underline{\sigma};\, l_t,\, d_t,\, e_t)$ and $\Omega_t^r \equiv \mathit{\Omega}(\overline{\sigma};\, l_t,\, d_t,\, e_t)$ respectively. Anything that would make $\Omega_t^r - \Omega_t^s$ go positive will trigger a risk-taking episode. Equation (25) specifies $\mathit{\Omega}(\sigma_t;\, l_t,\, d_t,\, e_t)$ for all values of σ_t, where it will be recalled that

$$\varepsilon_{t+1}^* = -\frac{Q_t}{\sigma_t}\left[R_{t+1}^s - R_t^d\left(1-\gamma_t\right)\right] \tag{38}$$

is the realization of a bank's idiosyncratic shock below which its net worth is negative, and $G(\varepsilon_{t+1}^*)$ is the probability that the bank will fail. Implicit in the formulation of the banker's problem, (23), is the fact that $G'(\varepsilon_{t+1}^*) > 0$ and $G(\varepsilon_{t+1}^*) \to 0$ as $\varepsilon_{t+1}^* \to -\infty$.

For purely expositional purposes, we will suppose that $\underline{\sigma} = 0$ and $\overline{\sigma} = 1$ in this section. With these simplifications, (25) implies

$$\Omega_t^s = \mathbb{E}_t\left[\psi_{t,t+1}\left(R_{t+1}^s - R_t^d\left(1-\gamma_t\right)\right)\right] l_t^s - \gamma_t l_t^s \quad \text{and} \tag{39}$$

$$\begin{aligned} \Omega_t^r = \mathbb{E}_t\Bigg[\psi_{t,t+1}\Bigg(\left(R_{t+1}^s - R_t^d\left(1-\gamma_t\right) - \frac{\xi}{Q_t}\right)\left(1 - G(\varepsilon_{t+1}^*)\right) + \\ \frac{\tau_t}{Q_t\sqrt{2\pi}}\exp\left(-\left(\frac{\varepsilon_{t+1}^* + \xi}{\tau_t\sqrt{2}}\right)^2\right)\Bigg)\Bigg] l_t^r - \gamma_t l_t^r, \end{aligned} \tag{40}$$

where it will be recalled that

$$R_{t+1}^s = \alpha\left\{\frac{A_{t+1}}{Q_t}\left(\frac{H_{t+1}^s}{K_{t+1}^s}\right)^{1-\alpha} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right\}. \tag{41}$$

What might turn $\Omega_t^r - \Omega_t^s$ positive, triggering a risk-taking episode? The obvious culprit is the interest rate spread $R_{t+1}^s - R_t^d (1-\gamma_t)$. An expected narrowing of this spread will decrease Ω_t^s more than Ω_t^r since $1 - G(\varepsilon_{t+1}^*)$ is less than one in the risk-taking regime. Moreover, a narrowing of the spread has a secondary effect on Ω_t^r that is a little more subtle: (38) implies that ε_{t+1}^* will rise. The presence of ε_{t+1}^* (instead of $-\infty$) in the bank's expected dividends, (23), represents the value of limited liability to banks. Idiosyncratic shocks below this cut-off point cannot lower the bank's expected (discounted) net worth. An increase in ε_{t+1}^* would enhance the value of the shield of limited liability and increase Ω_t^r.[27] Note finally that if a risk-taking episode is triggered, there will be a jump in σ, and therefore a further jump in ε_{t+1}^*.

So, what might narrow the interest rate spread and provoke a risk-taking episode? There are a number of possibilities. Perhaps the most obvious would be a fall in the expected return on safe assets; for example, an expected fall in TFP could trigger a risk-taking episode. Two parameters in (40) are also of interest. An increase in the standard deviation of the idiosyncratic shock, τ_t, will raise Ω_t^r since it increases the upside potential of the risky asset (while the downside potential is unchanged because of limited liability). The second parameter is the expected value of the risky firm's idiosyncratic shock, $-\xi$; ξ is the average penalty for investing in the risky asset. A fall in this parameter would also raise Ω_t^r.

Note also that a loosening of the capital requirement, γ_t, would decrease the interest rate spread and could trigger a risk-taking episode. A loosening of the capital requirement allows the bank to fund more of its loans with deposits; this reduces the cost of banking and allows the bank to keep less skin in the game. The bank expands its lending and switches to risky loans. And note finally that a dynamic capital requirement could hold $\Omega_t^r - \Omega_t^s$ constant at its steady-state value; banks would never leave the safe equilibrium.

The intuitive exposition just given relied upon two simplifying assumptions—one made explicit, and the other implicit—that must now be undone. The explicit assumption was that $\underline{\sigma} = 0$ and $\overline{\sigma} = 1$. In the numerical analysis that follows, $\underline{\sigma}$ is set equal to 0.01 and $\overline{\sigma}$ is set equal to 0.99; in equilibrium, there must be both safe and risky loans (and firms) so that we can track safe and risky capital across regimes. The implicit assumption was that a bank could observe both Ω_t^r and Ω_t^s, and then choose its loan portfolio accordingly. But, we cannot have both Ω_t^r and Ω_t^s in equilibrium. If we are not in a risk-taking episode, we have Ω_t^s, and Ω_t^r is an off-equilibrium object; during a risk-taking episode, we have Ω_t^r, and Ω_t^s is an off-equilibrium object.

However, there is an equilibrium spread in asset returns—whose evolution is closely

[27] It is hard to see these results in (40) without investigating a number of special cases, some involving the absolute value of $\varepsilon_{t+1}^* + \xi$. These special cases are relegated to Appendix C.

related to $\Omega_t^r - \Omega_t^s$—that we can track:

$$S_t \equiv \mathbb{E}_t \left[R_{t+1}^{e,r} - R_{t+1}^{e,s} \right]. \tag{42}$$

S_t is the expected spread between the returns on risky and safe equity. Because of our minimum scale assumptions, a small amount of risky loans will be extended in the safe regime, and conversely, a small amount of safe loans will be extended in the risky regime; so, the returns on equity are equilibrium objects. In a risk-taking episode, S_t turns positive. Once the episode is over, the spread turns negative.[28]

4 Dynamic Capital Requirements

Before discussing dynamic capital requirements, we build some intuition for how changes in capital requirements affect our model economy. The next two sections show that sufficiently large increases and decreases in capital requirements have asymmetric effects on the decisions of banks, economic outcomes, and welfare. In the subsequent sections, we introduce our no-failure capital policy and discuss its global optimality for a static version of our model.

4.1 An Increase in Capital Requirements

Figure 3 shows the effects of a one percentage point increase in the capital requirement, γ_t; this increase is implemented through a shock that follows an autoregressive process of order 1 with a persistence parameter of 0.9. The increase forces banks to shift the funding mix from deposits to equity; this shift increases the cost of funding a given amount of loans since deposits will be held by the households at a lower rate of return because of their liquidity value.

Note that the Modigliani-Miller Theorem does not hold in our model since, once again, deposits are valued for their transaction services. So, even though the economy stays in a

[28] There is a simple relationship between S_t and $\Omega_t^r - \Omega_t^s$ when computing Ω_t^r and Ω_t^s conditional on, respectively, the risky and safe loans actually extended (rather than the *desired* amount of loans). In that case, $S_t \equiv \mathbb{E}_t \left[R_{t+1}^{e,r} - R_{t+1}^{e,s} \right] = \frac{\Omega_t^r}{E_t^r} - \frac{\Omega_t^s}{E_t^s}$. The thought experiment by which a banker compares the expected dividends for a desired level of loans is intuitive, but we solve the model by referring to the Lagrange multipliers on the non-negativity constraints for safe and risky loans. When extending safe loans leads to higher expected dividends, a banker would want to short-sell risky loans, turning the corresponding Lagrange multiplier positive; analogously, when extending risky loans leads to higher expected dividends, a banker would want to short-sell safe loans. These two conditions allow us to determine which regime applies in any period more easily than attempting to construct $\mathbb{E}_t \frac{\Omega_{t+1}^r}{E_t^r}$ and $\mathbb{E}_t \frac{\Omega_{t+1}^s}{E_t^s}$, whose computation requires taking a stand on the entire path of future actions.

safe equilibrium, tighter capital requirements can have real effects on the macroeconomy.

More precisely, an increase in the capital requirement acts like a tax hike on banks. Households, who own the banks, are effectively poorer. They cut back on consumption, and since labor is inelastically supplied, their savings increase correspondingly. But under our calibration, the movements in consumption, investment, and output are tiny, as can be seen in Figure 3. The real side of the economy is hardly affected.

By contrast, the effects in the financial sector are sizable and can affect household utility. First and foremost, the increase in equity funding reduces the bank's demand for deposits, and the deposit rate falls. Moreover, the increase in household savings pushes up the supply of deposits, which reinforces the decrease in the deposit rate. Deposits make up roughly 90 percent of bank funding in our calibration. Somewhat surprisingly, the increase in capital requirements and the subsequent fall in the deposit rate end up *reducing* the cost of banking.[29] However, the large drop in deposits, coupled with the (almost imperceptible) fall in consumption, decreases household utility, as can be seen in the last panel in Figure 3.[30]

Over time, these movements reverse themselves. The capital requirement falls, and deposits recover. The capital stock falls, increasing the marginal product of capital and R^s, which pushes Ω^s up relative to Ω^r. The economy reverts to its steady state.

4.2 A Decrease in Capital Requirements

The dashed lines in Figure 4 show responses to a shock that lowers γ by one percentage point. The shock is equal in magnitude but opposite in sign relative to the one considered in the previous section—the effects of that shock are shown again by the solid lines to emphasize the asymmetry.

In response to the decline in capital requirements, deposits rise and bank equity falls. The steady state capital requirement, as described in Section 3, is set to 7.8 percent. The minimum capital requirement consistent with a safe regime varies with aggregate shocks, but in the steady state it takes a value of 7.21 percent—in other words, the model calibration implies a modest steady-state buffer sized at 0.59 percentage points. A one percentage-point shock more than depletes this buffer and plunges the economy into the risky regime.

On average, risky firms produce less output since a risky firm's idiosyncratic shock has a negative expected value; so, output and income fall substantially.[31] Consumption and

[29] Begenau (2020) also finds that an increase in capital requirements can reduce the cost of bank funding and increase lending.

[30] Welfare is calculated as the present discounted value of utility at a given point in time; it moves as the state variables change.

[31] Put another way, some of the risky loans fail, destroying bank equity and increasing the taxes necessary to insure deposits. So, output and income fall.

investment also fall. Because of the discontinuous hit to the economy's productive capacity, welfare takes a sizable hit.

To sum up, positive and negative shocks to the capital requirement have asymmetric effects on the economy, and they are not the mirror images found in linear models. For a tightening of capital requirements, what happens in the financial sector mostly stays in the financial sector, but welfare still falls from a reduction in deposit balances. By contrast, loosening capital requirements may trigger an excessive risk-taking episode with a more than proportionate fall in output, consumption, and welfare.

4.3 A Candidate Optimal Policy

In line with the discussion in the preceding section, our candidate optimal policy sets capital requirements $\{\gamma_t^*\}_{t=0}^{\infty}$ at the lowest level necessary to prevent falling into the risk-taking regime—given realizations of the shocks—at any date t. We dub this policy "no-failure capital requirements."

As illustrated in Section 4.1, a higher capital requirement in period t_k would lead to welfare losses from the reduced amount of liquidity services without altering risk-taking incentives. The decrease in the capital requirement involves an output loss of about ξK from making risky loans, but it may increase the liquidity services that enter into household utility. The trade-off between these two considerations determines the impact on welfare. For a small decrease in capital requirements, the former consideration is more important. Why? Since banks jump to the risky equilibrium, the marginally lower capital requirement entails a discrete, disproportionate drop in welfare stemming from the discrete drop in output, as illustrated in Section 4.2. By contrast, the welfare change associated with increased liquidity provision by banks stemming from marginally lower capital requirements is proportional to the change in capital requirements.

Our reasoning above establishes that the no-failure policy is locally optimal. We offer two checks beyond local optimality. In the next section, we show that this policy is globally optimal in a static setting. Furthermore, in a dynamic setting, we can compare the performance of our candidate optimal rule against different classes of rules. Our tests fail to reject global optimality within the class of linear policy rules, as we discuss in Section 6.2.

4.4 The Optimal Steady-State Capital Requirement

We can investigate the optimality of our candidate policy in the steady state by direct evaluation of a grid of values for the capital requirement. The results are shown in Figure 5.

The vertical dashed line in each panel demarcates the border between the risky regime, on the left of the line, and the safe regime, on the right. Starting with the safe regime, the top panel of the figure shows that welfare peaks at a capital requirement just large enough to prevent the economy from falling into the risky regime. By inspection, this finding makes our candidate policy globally optimal in this static setting. For higher values of the capital requirement, deposits drop as banks need to shift the financing mix away from deposits. Consumption moves down progressively but imperceptibly, as does the capital stock of firms (equivalent to outstanding credit in our model). The drop in consumption and deposits is monotonic for values of the capital requirement to the right of the regime shift. Accordingly, the drop in welfare is also monotonic in that region.

For values of the capital requirement that let the risky regime prevail, the interactions among multiple forces are more complex. Starting from the lowest capital requirements shown in the figure, marginal increases in this requirement continue to reduce deposits and their liquidity value monotonically; by raising the cost of credit, increases in capital requirements buffet the equilibrium level of physical capital monotonically. But the effect of declines in physical capital on consumption is not monotonic. At first, consumption rises as a lower stock of physical capital reduces the amount of investment needed to replenish depreciated capital. But as capital requirements surpass roughly 6.2 percent, consumption starts dropping. At that point, as physical capital becomes scarcer, further reductions buffet output more and more, and from there, consumption. In other words, with a positive second derivative of capital in the production function, reductions in capital eventually lead to such large cuts in production and consumption that this effect prevails over offsetting effects on consumption from reductions in investment.

The considerations for welfare reflect the interplay of the utility value of both consumption and deposits. For capital requirements between roughly 5.2 and 6.2 percent, marginal increases in the capital requirement reduce the utility value of deposits more than they increase the utility value of consumption. Accordingly, welfare has a local peak at about 5.2 percent.

5 Illustrating the Reaction of Optimal Dynamic Capital Requirements to Aggregate Economic Shocks

In this section, we show how our candidate optimal rule for capital requirements, γ_t, reacts to three shocks that have different cyclical implications for GDP and credit. Specifically, we show that optimal capital requirements can increase in a recession or a boom, or may adjust to prevent a banking crisis in response to shocks that leave little imprint on GDP. We also show that, depending on the source of shocks, our candidate optimal rule implies different patterns of correlation with the credit-to-GDP ratio.

5.1 A Contractionary TFP Shock

TFP shocks have played a major role in RBC modeling. Figure 6 illustrates the effects of a negative one-standard-deviation TFP shock. We consider three alternative rules for capital requirements. In each panel, the solid lines show responses under our candidate optimal rule for capital requirements. The dashed lines show responses under the estimated static buffer. And the dot-dashed lines show responses under a rule that suppresses the static buffer and simply keeps capital requirements constant at the lowest level that would prevent falling into the risky regime in the steady state. This rule is optimal at the steady state but performs poorly in the face of shocks. We use it to illustrate the perils of a suboptimal policy that does not prevent a switch to the risky regime.

We begin with the case of fixed capital requirements with no buffer. Since the shock is auto-correlated, today's TFP shock lowers the expected marginal productivity of capital for the next period, and thus the expected return on safe assets. As explained in Section 3.3.1, as risky projects become relatively more attractive, the equilibrium switches to a risky regime. The term $R^{e,s}_{t+1}$ falls, and the spread between risky and safe projects turns positive. Risky firms produce less output on average, leading to bank failures; so, output and income fall substantially, as does consumption.

Over time, the TFP shock dissipates and the process described above reverses itself. The falling capital stock raises the marginal productivity of capital, the return on safe assets, and the price of investment. S_t falls, and jumps negative after σ_t drops to its lower bound, and the economy jumps back to a safe equilibrium. The credit-to-GDP ratio rises and then, midway, starts to fall.

Next, we turn to our no-failure rule for capital requirements. In this case, the responses are denoted by the solid lines in Figure 6. The rule sets capital requirements just tight enough to keep safe loans attractive; as we have seen, any higher would unnecessarily deprive households of the deposits that they value. γ_t jumps on impact and falls back to its steady-state value as the TFP shock dissipates. Notice that the change in capital requirements is very small, just a fraction of a basis point.

While the planner's policy avoids risk-taking episodes, it cannot undo the damage done by the negative TFP shock itself. The shock lowers the household's net worth, and the household responds by decreasing consumption, as familiar from the RBC literature.

For use in Section 6, we also track the credit-to-GDP ratio. It rises and falls, as under our calibration, bank loans decrease more quickly than GDP. To set the stage for the analysis of simple rules, including the Basel III CCyB prescriptions, this is a shock for which the response of the credit-to-GDP ratio tracks the optimal response of capital fairly closely, but they are just on very different orders of magnitude.

Finally, notice that the responses under the calibrated static buffer for bank capital are indistinguishable from the response under the optimal rule. The only daylight appears for the expected spread between the returns from risky and safe projects. The optimal policy keeps those returns aligned, whereas the calibrated buffer forces more skin in the game than necessary, opening up a spread, as shown in panel 5.

Takeaways: The no-failure policy avoids a banking crisis in the face of an economic contraction caused by a negative TFP shock with an increase in capital requirements. Following a TFP shock, the responses of optimal capital and of the credit-to-GDP ratio are several orders of magnitude apart.

5.2 An Expansionary Investment Technology Shock

Here we study a positive η_t shock in the equation for net investment, (11). The innovation to the shock process is sized at one standard deviation based on the estimate from Section 3. Figure 7 illustrates the effects of this shock. Once again, the dashed-dotted lines show responses under a policy that keeps capital requirement unchanged from the lowest level that would prevent excessive risk-taking in the steady state—labeled no capital buffer.

This shock was not considered in Section 3.3.1, but its effects are readily translatable to the discussion there. A positive shock to investment in period t increases the supply of capital next period, K_{t+1}, lowering the expected marginal product of capital and the expected return on the safe asset. The expected return on safe equity falls, the spread between risky and safe returns turns positive, and an associated banking crisis begins, even though the shock itself is expansionary.

Note that the expected return on safe equity is short lived. To see why, note that the decrease in the marginal product of capital causes the price of capital, Q_{t+1}, to fall, raising the return on safe loans in period $t + 2$. However, the damage is already done; the risk-taking episode has already been triggered, as documented by the jump in S_t. The risky firms produce less output on average, and output and consumption fall. From here on, the story is much the same as before. The investment shock decays over time and the process gradually reverses itself. Note that there is an upward spike in the expected return on safe loans when the economy jumps back to a safe equilibrium.

The solid lines illustrate what would happen under our candidate optimal policy for γ_t.

The planner raises the capital requirement just enough to offset the switch to excessive risk-taking. Consumption and investment rise more in this case since there are no bankruptcies and equity losses to lower household income. Once again, there is little daylight between the responses under the optimal policy and the responses under the calibrated static buffer, shown by the dashed lines.

In response to this shock, the credit-to-GDP ratio may not move monotonically. Under our no-failure policy, it drops on impact, then it rises above its steady state, before falling again. By contrast, under the same policy, capital requirements rise and then fall monotonically.

Takeaways: The no-failure policy avoids a banking crisis in the face of an economic boom caused by an investment technology shock with an increase in capital requirements. Under this rule, there is no time-invariant contemporaneous correlation between capital requirements and the credit-to-GDP ratio; the correlation pattern also varies depending on which shock or combination of shocks is affecting the economy.

5.3 A Shock that Increases the Volatility of Risky Returns

As discussed in Section 3, τ, the mean standard deviation of the idiosyncratic shock affecting risky firms is 4.6%. Our volatility shock increases the standard deviation by 17 basis points, after which it follows the estimated autoregressive process of order 1. As explained in Section 3.3.1, an increase in volatility raises the expected return on risky loans, since it enhances the upside potential of risky loans while the downside risk is protected by limited liability.

Figure 8 illustrates the economic consequences of this volatility shock. As before, the dashed lines show what would happen if γ_t were to be held constant with no buffer. The shock entices banks to switch to risky loans, some of which will fail, increasing taxes and destroying bank equity. The story that follows is by now familiar. Consumption and investment fall. Eventually, the shock dissipates and the falling capital stock raises R^s enough to make safe loans attractive again.

As the solid lines illustrate, our no-failure policy raises capital requirements just enough to eliminate the excessive risk-taking. Under this policy, there is no change in the expected return on safe equity or on S_t, and the shock has no discernible effect outside financial markets. Once again, as shown by the dashed lines, the responses under the calibrated buffer hug the responses under the optimal policy.

Finally, returning to the response of the credit-to-GDP ratio, the optimal Ramsey policy for capital requirements leaves it essentially unchanged in response to the volatility increase.

Takeaways: The no-failure policy can neutralize the effects of shocks that affect the desirability of risky projects by raising capital requirements leaving little to no imprint on GDP and the credit-to-GDP ratio. This is further evidence that the correlation pattern between optimal capital requirements and the credit-to-GDP ratio is influenced by the underlying shocks.

6 Simple and Implementable Rules for Capital Requirements

Section 5 discussed the effects of some key sources of shocks in isolation. In practice, policymakers face a much more difficult challenge: as the economy faces a multiplicity of shocks, all occurring at the same time, policymakers have to consider the full stochastic structure of the economy. In our model, we can deploy our no-failure policy when the economy is hit by a full constellation of shocks, but it is implausible to think that policymakers would be able to implement it. So, in this section, we consider policy rules in which the capital requirement responds to key indicators of the state of the business or credit cycles without conditioning on precise knowledge of each shock at all points in time.

We focus here on simple rules that respond to only one variable, but we also allow the rules to retain a static buffer over the amount of bank capital necessary to avoid switching into the risky regime in the steady state. The Basel III cyclical buffer, which emphasizes the credit-to-GDP ratio, will be of particular interest. We will optimize the coefficients of the rules to maximize welfare. In practice, once optimized, the dependence on cyclical indicators is all but shut off, so that the simple rules are hard to distinguish from a simple static buffer.

6.1 Evaluating Simple Rules

We start this exploration by focusing on the Basel rule and showing that it does little to improve economic outcomes. The Basel III prescription is to tighten or relax capital requirements in line with changes in the credit-to-GDP ratio.[32]

As shown in Table 4, the optimized coefficient on the credit-to-GDP ratio is small in magnitude, a mere 0.000035, limiting the variation in the capital requirement. The standard deviation of γ_t under this rule is below one basis point, well below the variation under the

[32] The Basel III guidance on the CCyB recommends detrending the credit-to-GDP ratio with a Hodrick-Prescott filter with a parameter set to 400,000, implying a trend very close to linear. Since our model is stationary, there is little difference between the ratio in deviations from its steady state and this gap. For simplicity, we focus on a rule in terms of the former, where the ratio itself is defined as $\frac{L_t^s+L_t^r}{4(Y_t^s+Y_t^r)}$. The annualization of the quarterly GDP flow follows standard practice.

best performing no-failure rule, as can be seen from the last row of the table. The average failure rate is only slightly higher than for the optimized buffer (shown in a lower row of the table), but this is only because, essentially, the optmimization of the coefficients makes the rule indistinguishable from an optimized static buffer. As a result, the loss relative to a fully optimal rule remains very small, about 2 basis points of permanent consumption.[33]

We emphasize that the welfare costs of this simple rule is so small merely because the optimization of the rule's coefficients eliminated any meaningful cyclical variation. The table stops short of showing that it is also possible to do more harm than good with a simple rule. For instance, consider an alternative rule that raises the capital requirement 1 percentage point for each percentage point increase in the credit-to-GDP ratio above its steady state level (i.e., the slope coefficient is 1). Under such a constrained rule, the optimal buffer would rise from about 0.85 percent to 6.85 percent. Correspondingly, the welfare loss would rise from about 2 basis points to nearly 1 percent of consumption, a large welfare loss by the standard of macro models. And of course, the welfare loss would be even larger without re-optimizing the buffer, but the model becomes so unstable that we have trouble quantifying it.

Much the same reasoning applies when considering rules optimized to respond to GDP, or to the expecting banking spread, equivalent to a net interest margin in our model. In fact, even when considering optimized rules depending on any other state variable from the model, we did not have much better luck. Apart from rules that respond to only one indicator, we considered rules dependent on multiple indicators, such as the one proposed by Davydiuk (2017). That rule includes three cyclical terms: a credit gap, a GDP gap, and a liquidity premium. We also allow for a static buffer and optimize all the rule coefficients for our model. Other than the static buffer, all the other coefficients become vanishingly small when optimized. The intuition for this result is the same as for the other rules: There is no stable dependence between the welfare-maximizing setting capital requirements and aggregate indicators of the state of the business or credit cycle in the face of the array of shocks of our model.

We conclude this section by highlighting that none of the simple rules, including the optimized static buffer, eliminates bank failures. We develop intuition for this finding with the results illustrated in Figure 9. The top panel shows the level of overall welfare. In line with the results offered in Table 4, the welfare-maximizing static buffer is sized at about 85 basis points. The next two panels show, respectively, the contribution to welfare of the liquidity services of deposits and the contribution of consumption. A higher capital buffer reduces welfare from deposits, even if it frees up resources to sustain a higher consumption

[33]The consumption equivalent variation is derived in Section D of the appendix.

level by reducing the failure rates of banks, as shown in the bottom panel. Higher buffers could drive the failure rate to zero, but the optimal buffer balances allowing infrequent bank failures with obtaining a higher liquidity value from deposits.

6.2 Another Check on the No-failure Rule

Returning to Table 4, the remaining rules are not implementable because of their informational requirements, but we offer them as a proof of concept. In line with our claim in Section 4 that knowledge of the shocks is important in charting the appropriate response of capital requirements, once we optimize a rule that responds to all shock processes (including the current innovations), and the lagged value of the capital requirement, γ_{t-1}, the rule becomes nearly identical to the no-failure rule—the bank failure rate drops to zero and the static buffer can be lowered to a mere 1 basis point. Correspondingly, the consumption equivalent variation for this rule is vanishingly small, below $\frac{1}{10}$ of a basis point of permanent consumption.

Once we optimize a rule that not only responds to all shocks processes but also includes all the state variables, we replicate our candidate optimal rule—the correlation between the capital requirements set by these two alternative specifications of the rule is 1. Accordingly, there is no daylight between the last two rows of the table. Having restarted the optimization of this comprehensive linear rule from a dispersed set of points and having failed to improved on the no-failure rule, we take the no-failure rule to be optimal in the class of linear rules with constant coefficients.

6.3 Volatility of Optimal Dynamic Capital Requirements

Table 4 shows that the volatility of the capital requirement, γ_t, is higher for the optimal rule than for the optimized simple rules that respond to alternative key aggregate indicators. Nonetheless, even for the optimal rule, the standard deviation of the capital requirement is just below a modest one half of a percentage point.

Apart from the volatility of aggregate shocks, three key parameters affect the volatility of γ_t: they are τ, the mean of the standard deviation of the idiosyncratic technology shock, ξ the average penalty from financing risky firms; and ς_d the inverse of the interest rate elasticity of the household's supply of bank deposits. An increase in τ, or a decrease in ξ, require a larger adjustment in capital requirements under the no-failure rule but no change in their cyclical properties. These results may not be too surprising, since these parameter changes make risky loans more attractive.

7 Conclusion

In our model, bank risk-taking is endogenous, and the temptation to take excessive (or socially inefficient) risk is enabled by limited liability and government deposit insurance. Both macroeconomic and market-volatility shocks can trigger bouts of excessive risk-taking by lowering the expected return on safer investments. Capital requirements can eliminate that temptation by forcing banks keep more skin in the game, but this may come at the cost of limiting liquidity-producing deposits.

Our benchmark rule sets capital requirements at the lowest level compatible with excluding bank failures. Direct optimization of a linear rule that responds to all the state variables of our model yields capital requirements that are perfectly correlated with those of our benchmark rule. This finding is consistent with our benchmark rule being optimal in the class of linear rules.

The benchmark no-failure rule is not implementable as it requires knowledge of the full constellation of shocks that drive the economy. This rule raises capital requirements in booms or busts, depending on the underlying shocks. And the same rule raises capital requirements in response to an increase in market volatility that has little consequence for the business cycle. Such informational requirements are daunting, even in our stylized model with only two projects that banks can finance. Moving beyond our model, regulators would have to keep track of expected relative returns for a myriad of possible projects and be able to track the effects of a plethora of shocks.

It is tempting to look for key market indicators that might point the way to appropriate changes in the capital requirement. However, we show that popular candidates, such as growth in the credit-to-GDP ratio, are unlikely to be reliable.

Fortunately, a small static buffer—slightly higher than the optimal steady-state capital requirement—avoids the Wile E. Coyote moments and achieves welfare levels close to the optimal linear rule. Some finely tuned policy rules, such as a rule following the Basel III guidance on the setting of countercyclical capital buffers, may sound sensibly grounded in empirical regularities but turn out to do more harm than good in our model.

Recent work is taking up our challenge to devise simple and implementable rules that do better than a static capital buffer. In this vein, Muñoz and Smets (2025) find that a rule that responds to net interest margins can improve on a static buffer. However, they focus on few sources of shocks. We do not find that a rule that responds to interest margins does well in the context of our model when we allow for a plethora of shock sources. However, the model in Muñoz and Smets (2025) has frictions that ours does not capture. Going forward, we hope that our work can be a catalyst for theoretically consistent and empirically grounded analysis of the robustness of different rules. We see this type of work as essential for the Hippocratic Oath—first, do no harm.

References

Angeloni, I., & Faia, E. (2013). Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics*, *60*(3), 311–324.

Basel Committee on Banking Supervision. (2010). Guidance for national authorities operating the countercyclical capital buffer. Bank for International Settlements.

Begenau, J., & Landvoigt, T. (2022). Financial regulation in a quantitative model of the modern banking system. *The Review of Economic Studies*, *89*(4), 1748–1784.

Begenau, J. (2020). Capital requirements, risk choice, and liquidity provision in a business-cycle model. *Journal of Financial Economics*, *136*(2), 355–378.

Bennett, R. L., & Unal, H. (2015). Understanding the components of bank failure resolution costs. *Financial Markets, Institutions & Instruments*, *24*(5), 349–389.

Boissay, F., Collard, F., & Smets, F. (2016). Booms and banking crises. *Journal of Political Economy*, *124*(2), 489–538.

Bordalo, P., Gennaioli, N., & Shleifer, A. (2018). Diagnostic expectations and credit cycles. *The Journal of Finance*, *73*(1), 199–227.

Borio, C., & Lowe, P. (2002). Assessing the risk of banking crises. *BIS Quarterly Review*. Bank for International Settlements.

Borio, C., & Lowe, P. (2004). Securing sustainable price stability: Should credit come back from the wilderness? BIS Working Paper, No. 157.

Brunnermeier, M. K., & Sannikov, Y. (2014). A macroeconomic model with a financial sector. *American Economic Review*, *104*(2), 379–421.

Christiano, L., Motto, R., & Rostagno, M. (2014). Risk shocks. *American Economic Review*, *104*(1), 27–65.

Collard, F., Dellas, H., Diba, B., & Loisel, O. (2017). Optimal monetary and prudential policies. *American Economic Journal: Macroeconomics*, *9*(1), 40–87.

Davydiuk, T. (2017). Dynamic bank capital requirements. *SSRN Electronic Journal*.

Elenev, V., Landvoigt, T., & Nieuwerburgh, S. V. (2021). A macroeconomic model with financially constrained producers and intermediaries. *Econometrica*, *89*(3), 1361–1418.

Fair, R. C., & Taylor, J. B. (1983). Solution and maximum likelihood estimation of dynamic nonlinear rational expectations models. *Econometrica*, *51*(4), 1169–1185.

Faria-e-Castro, M. (2021). A quantitative analysis of the countercyclical capital buffer. ESRB Working Paper Series, No. 120.

Gertler, M., & Karadi, P. (2011). A model of unconventional monetary policy. *Journal of monetary Economics*, *58*(1), 17–34.

Gertler, M., & Kiyotaki, N. (2015). Banking, liquidity, and bank runs in an infinite horizon economy. *American Economic Review*, *105*(7), 2011–43.

Gertler, M., Kiyotaki, N., & Prestipino, A. (2020). A macroeconomic model with financial panics. *The Review of Economic Studies*, *87*(1), 240–288.

Gomes, J. F., Grotteria, M., & Wachter, J. A. (2023). Foreseen risks. *Journal of Economic Theory*, *212*, 105706.

Gorton, G., & Metrick, A. (2013). Securitization. *Handbook of the Economics of Finance*, *2*, 1–70.

Guerrieri, L., & Iacoviello, M. (2015). Occbin: A toolkit for solving dynamic models with occasionally binding constraints easily. *Journal of Monetary Economics*, *70*, 22–38.

He, Z., & Krishnamurthy, A. (2012). A model of capital and crises. *The Review of Economic Studies*, *79*, 735–777.

Jordà, Ò., Schularick, M., & Taylor, A. M. (2017). Macrofinancial history and the new business cycle facts. *NBER Macroeconomics Annual*, *31*(1), 213–263.

Malherbe, F. (2020). Optimal capital requirements over the business and financial cycles. *American Economic Journal: Macroeconomics*, *12*(3), 139–74.

Martinez-Miera, D., & Suarez, J. (2012). A macroeconomic model of endogenous systemic risk taking. CEPR Discussion Paper No. DP9134.

Mendoza, E. G. (2010). Sudden stops, financial crises, and leverage. *American Economic Review*, *100*(5), 1941–66.

Mian, A., Sufi, A., & Verner, E. (2017). Household debt and business cycles worldwide. *The Quarterly Journal of Economics*, *132*(4), 1755–1817.

Mian, A., Sufi, A., & Verner, E. (2020). How does credit supply expansion affect the real economy? the productive capacity and household demand channels. *The Journal of Finance*, *75*(2), 949–994.

Mishin, A. (2023). Dynamic bank capital regulation in the presence of shadow banks. *Review of Economic Dynamics*, *51*, 965–990.

Muñoz, M. A., & Smets, F. (2025). The positive neutral countercyclical capital buffer. Bank of England working papers, No. 1128.

Pancost, N. A., & Robatto, R. (2023). The effects of capital requirements on good and bad risk-taking. *The Review of Financial Studies*, *36*(2), 733–774.

Schularick, M., & Taylor, A. M. (2012). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. *American Economic Review*, *102*, 1029–61.

Van den Heuvel, S. J. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics*, *55*(2), 298–320.

Tables and Figures

Table 1: **Model Parameters**

Conventional Parameters			
Parameter	*Value*	*Interpretation*	
α	0.3	Capital share in production	
δ	0.025	Depreciation rate	
ϱ_c	1.1	Elasticity of substitution for consumption	
ς_d	1.1	Interest rate elasticity of supply of deposits	
Model-Specific Parameters			
Parameter	*Value*	*Interpretation*	*Explanation*
β	0.9968	Discount rate	Risk-free rate = 1.28%
γ	0.078	Capital requirement	Equity-to-asset ratio = 7.8%
ς_0	0.0298	Weight on liquidity in the utility function	Risk-free rate − deposit rate = 0.55%
$\underline{\sigma}$	0.01	Minimum risk that banks can take	Needed for numerical solution
$\overline{\sigma}$	0.99	Maximum risk that banks can take	Needed for numerical solution
τ	0.046	Mean std. dev. of idiosyncratic shock	Estimated by SMM
ξ	0.0012	Minus mean of idiosyncratic shock	Estimated by SMM
ϕ	0.17	Investment adjustment costs	Estimated by SMM
κ	0.76	Consumption habits	Estimated by SMM
Aggregate Shock Processes			
Shock	*AR(1) coef*	*Innovation Std. Dev.*	*Explanation*
TFP	0.96	0.0101	Estimated by SMM
ISP	0.63	0.0098	Estimated by SMM
Volatility	0.71	0.0017	Estimated by SMM

Note: See Section 3 for a description of the calibration strategy. For the shock processes, TFP refers to the total factor productivity shock, A_t; ISP refers to the investment-specific technology shock, η_t, and Volatility, refers to the shock to the volatility of the idiosyncratic technology shock, τ_t.

Table 2: **Second Moments Targeted: Data and Model Counterparts, 1980:Q1-2024:Q4**

	Data	Model	Model 5th perc.	Model 95th perc.
Var(GDP)	1.36	1.47	0.83	2.12
Corr(GDP,Investment)	0.88	0.98	0.97	0.99
Corr(GDP,Investment Price)	-0.09	0.14	-0.16	0.38
Var(Investment)	14.90	14.89	8.61	21.20
Corr(Investment,Investment Price)	0.06	0.08	-0.21	0.34
Var(Investment Price)	0.52	0.91	0.57	1.19
Autocorr(GDP)	0.90	0.90	0.85	0.93
Autocorr(Investment)	0.94	0.89	0.85	0.92
Autocorr(Investment Price)	0.92	0.85	0.80	0.88
Avg. bank default rate, %	1.52	1.52	0.52	2.68
Avg. GDP Gap with high default, %	-0.33	-0.33	-1.12	0.28

Note: The table reports the second moments targeted in the SMM procedure for the model calibration —variances, correlations, and autocorrelations at business-cycle frequencies. The model moments are computed from a simulated sample of 5000 observations. In addition to the second moments reported in the table, the SMM procedure targets the annualized average bank failure rate, and the average GDP gap in quarters when the bank failure rate exceeded 1 percent.

Table 3: **Variance Decomposition, 1980:Q1-2024:Q4**

	var(GDP)	var(invest.)	var(invest. p.)	var(credit/GDP)
TFP	96	94	12	56
ISP	0	2	88	42
Volatility	4	4	0	2

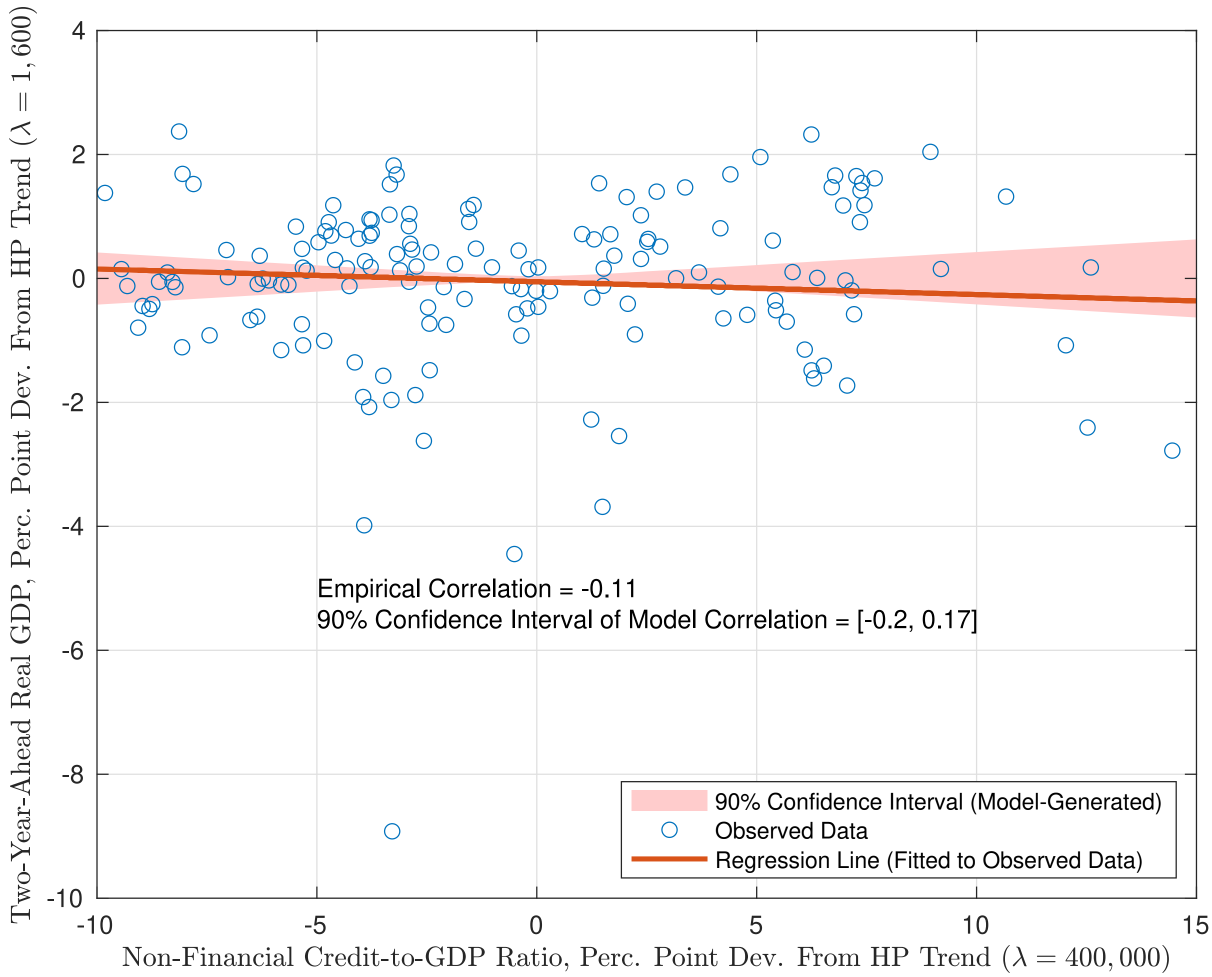
Note: The table shows the variance decompositions, in percent, for GDP, gross investment, the relative price of investment, and the credit-to-GDP ratio. TFP refers to the shock to total factor productivity, A_t; ISP refers to the shock to investment-specific technology, η_t, and Volatility, refers to the shock to the volatility of the idiosyncratic technology, τ_t.

Table 4: **Evaluating Alternative Rules for Capital Requirements**

	Slope	**Buffer** (percent)	**Std. Dev. of γ_t** (perc. point)	**Ave. Fail. Rate** (annualized percent)	**Average Welfare**	**Cons. Equiv. Variation** (percent)
Simple Rules						
Credit-to-GDP Ratio	-0.000035	0.842	0.008	0.276	-208.92	0.021
GDP Gap	0.000986	0.791	0.104	0.363	-208.92	0.021
Expected Banking Spread	0.051261	0.847	0.015	0.268	-208.92	0.021
Optimized Buffer		0.843	0	0.275	-208.92	0.021
Other Rules						
Davydiuk (2017) Rule	See Note	0.833	0.139	0.250	-208.92	0.020
All Shocks & γ_{t-1}	See Note	0.010	0.459	0	-208.85	0.000
All Shocks & State Variables	See Note	0.010	0.458	0	-208.85	0.000
No-Failure Rule		0	0.458	0	-208.85	0

Note: The table allows welfare comparisons for different simple rules for the capital requirement γ_t against the no-failure rule that sets bank capital requirements as just high enough to prevent bank failures. The simple rules include a static buffer on top of the steady-state optimal level of capital and other terms. The "Buffer" and "Slope" columns report, respectively, the buffer and the slope coefficient on the variable entering the rule optimized to maximize welfare. For example, for the row "Credit-to-GDP Ratio" the "Slope" column reports the optimized coefficient on the credit-to-GDP ratio. The column "Std. Dev. γ_t" reports the standard deviation of the capital requirement, γ_t. The column "Ave. Fail. Rate" reports average annualized failure rate—this is expressed as a ratio of the assets of failed banks to the assets in the banking sector. Finally, the column "Cons. Equiv. Variation" reports the consumption equivalent variation, i.e., the permanent change in consumption that would have to be offered with the simple rule in force to make the representative household indifferent between the simple and the no-failure rule. The rule in the rows labeled "Davydiuk (2017) Rule" responds to the credit-to-GDP ratio, GDP gap, and liquidity premium calculated as the difference between the return on safe loans and the return on deposits. The rules in the rows labeled "All Shocks & γ_{t-1}" and "All shocks & State Variables" respond, respectively, to: all shocks, their innovations, and lagged capital requirements; and to all shocks, innovations, and state variables (which include lagged capital requirements). The slope coefficients are too numerous to include here but can be accessed from the replication codes.

Figure 1: **Elevated Credit Predicts Lower GDP, Model and Data, 1980:Q1 2024:Q4**

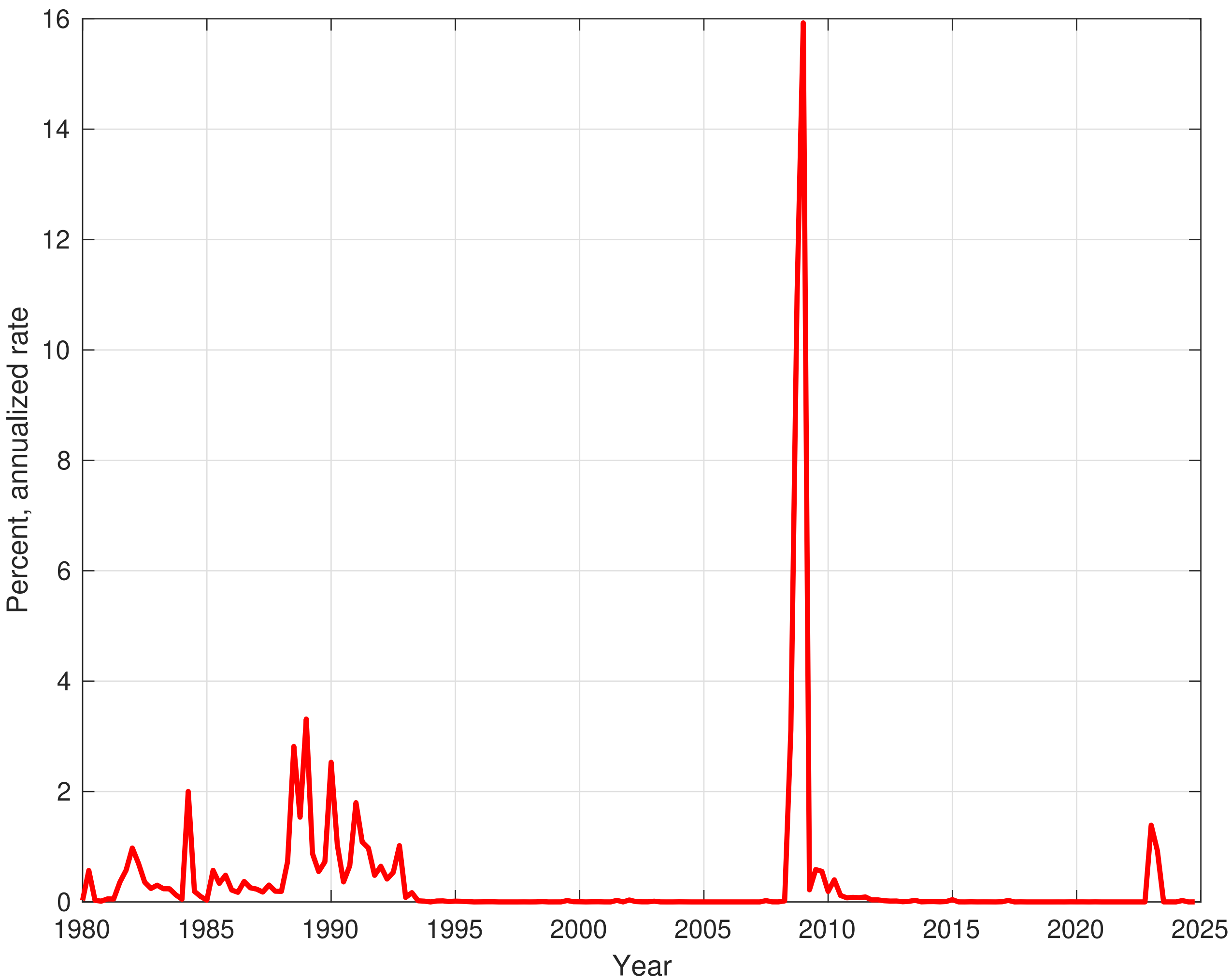


Source: Authors' calculations based on non-financial credit from the Bank of International Settlements and GDP data from the U.S. Bureau of Economic Analysis (NIPA).

Note: The figure plots the empirical and model relationships between the ratio of non-financial credit to GDP and real GDP two years ahead using data for the United States from the first quarter of 1980 through the fourth quarter of 2024. The open circles show the observed data. The solid line shows the regression between real GDP two years ahead and the credit-to-GDP ratio from observed data. The shaded area denotes a 90 percent confidence interval for the same regression slope coefficient using the model simulated data (1,000 samples with the same number of observations as the observed sample). The credit-to-GDP ratio was detrended with a Hodrick-Prescott filter using a coefficient of 400,000. Real GDP was detrended with a Hodrick-Prescott filter using a coefficient of 1,600.

Figure 2: **Bank Failure Rate, 1980:Q1 2024:Q4**

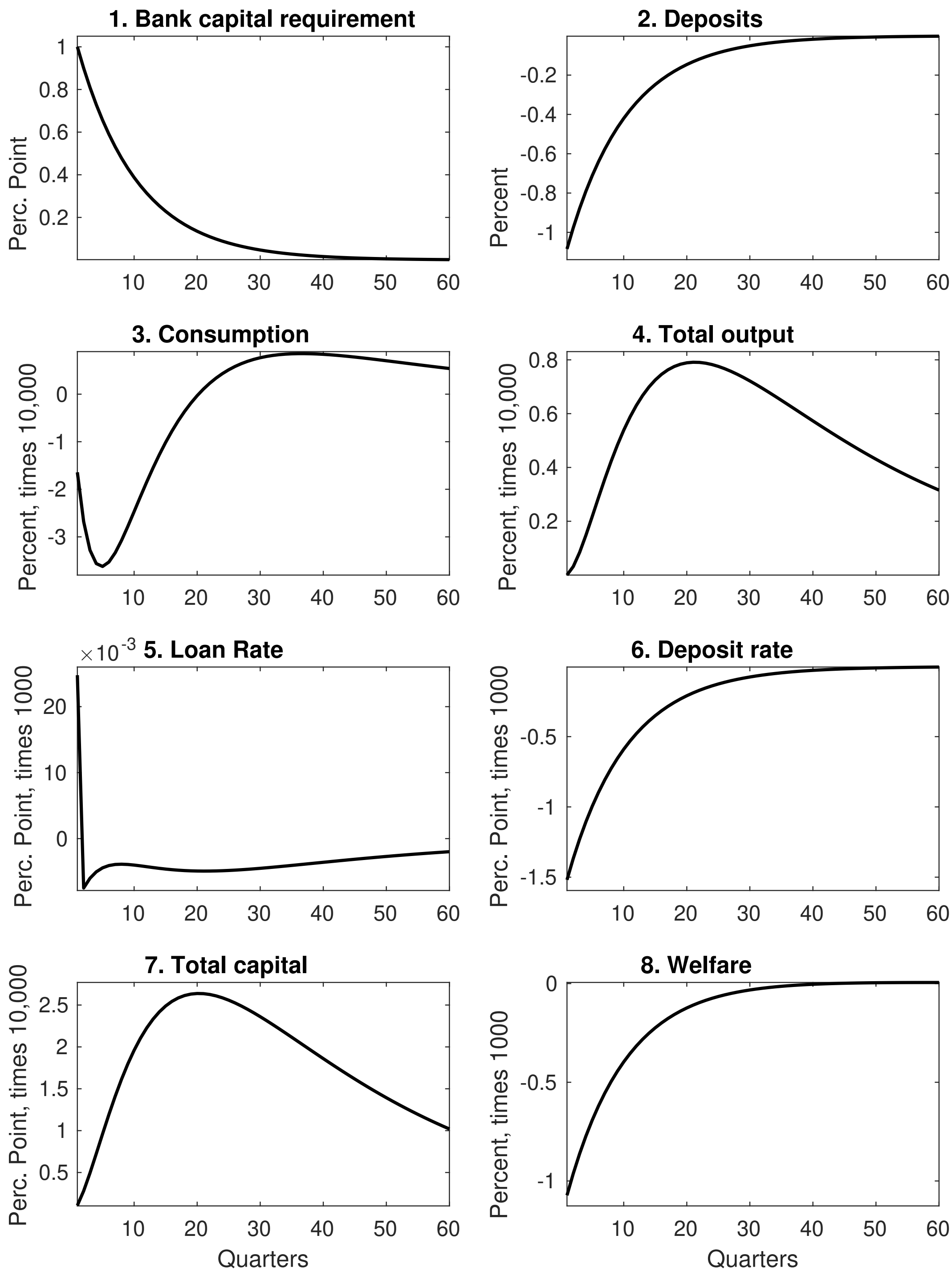
Annualized ratio of the assets of FDIC insured failed banks to total banking assets



Source: Authors' calculations based on FDIC list of failed banks, Call Report, and the H.8 Release of the Federal Reserve Board.

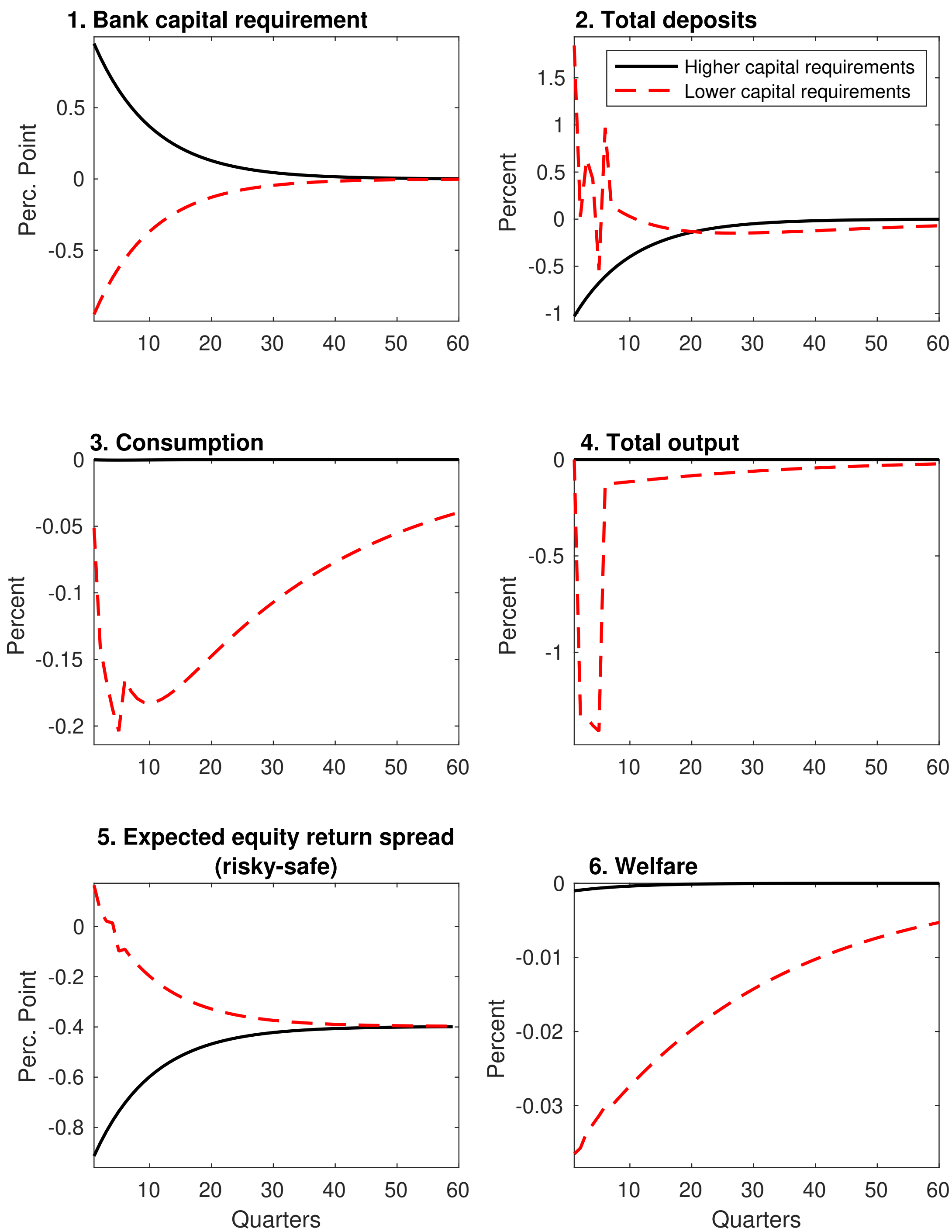
Figure 3: **A Temporary Increase in the Bank Capital Requirement**

Higher capital requirements reduce deposits but leave little imprint on the rest of the economy



Note: This figure plots the responses of variables (in deviation from the steady state) to a one percentage point increase in the capital requirement. The shock follows an AR(1) process with a persistence parameter of 0.9.

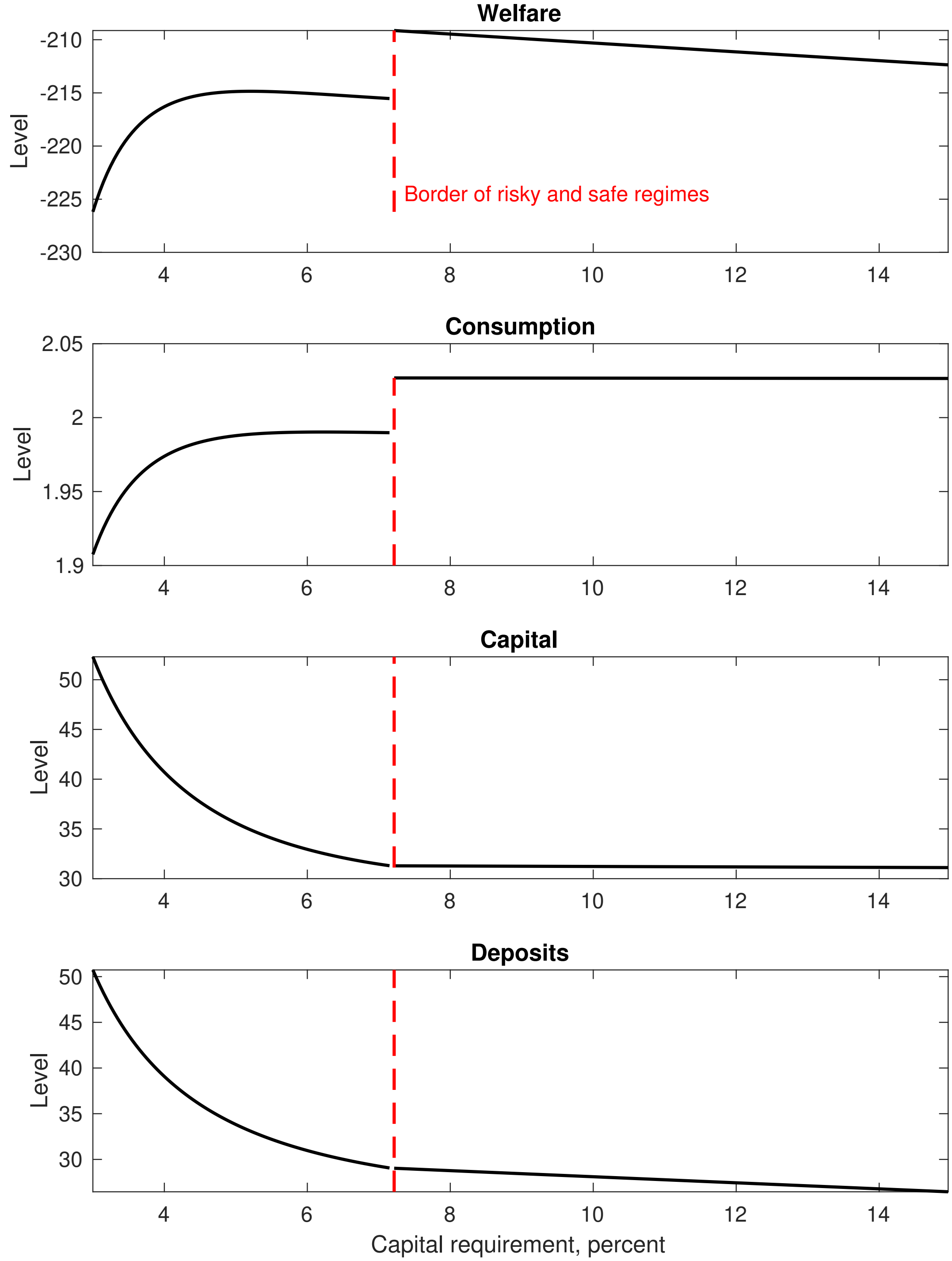
Figure 4: **Asymmetric Effects of Higher and Lower Capital Requirements**



Note: This figure shows the responses of variables (in deviation from the steady state) to a one percentage point change in the capital requirement. The shock follows an AR(1) process with a persistence parameter of 0.9. The solid line shows the responses to a one percentage point rise in the capital requirement. The dashed line shows the responses to a one percentage point fall in the capital requirement.

Figure 5: **Alternative Steady States Depending on Bank Capital Requirements**

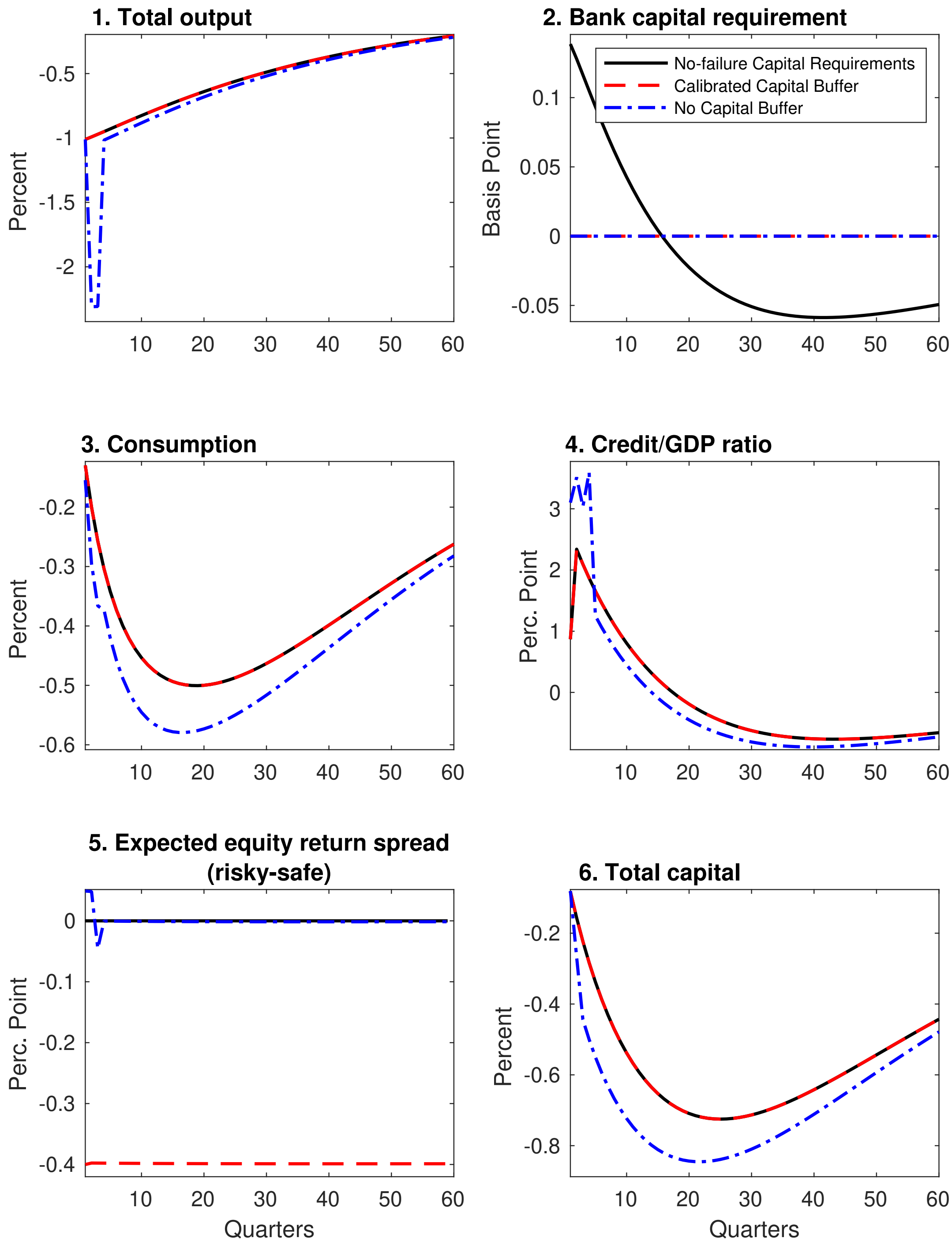
The smallest capital requirement that prevents bank failures maximizes welfare.



Note: For capital requirements to the left of the dashed vertical line, the model plunges in the risk-taking regime with bank failures. No bank failures occur in the steady state for capital requirements to the right of the dashed vertical line.

Figure 6: **A Negative TFP Shock**

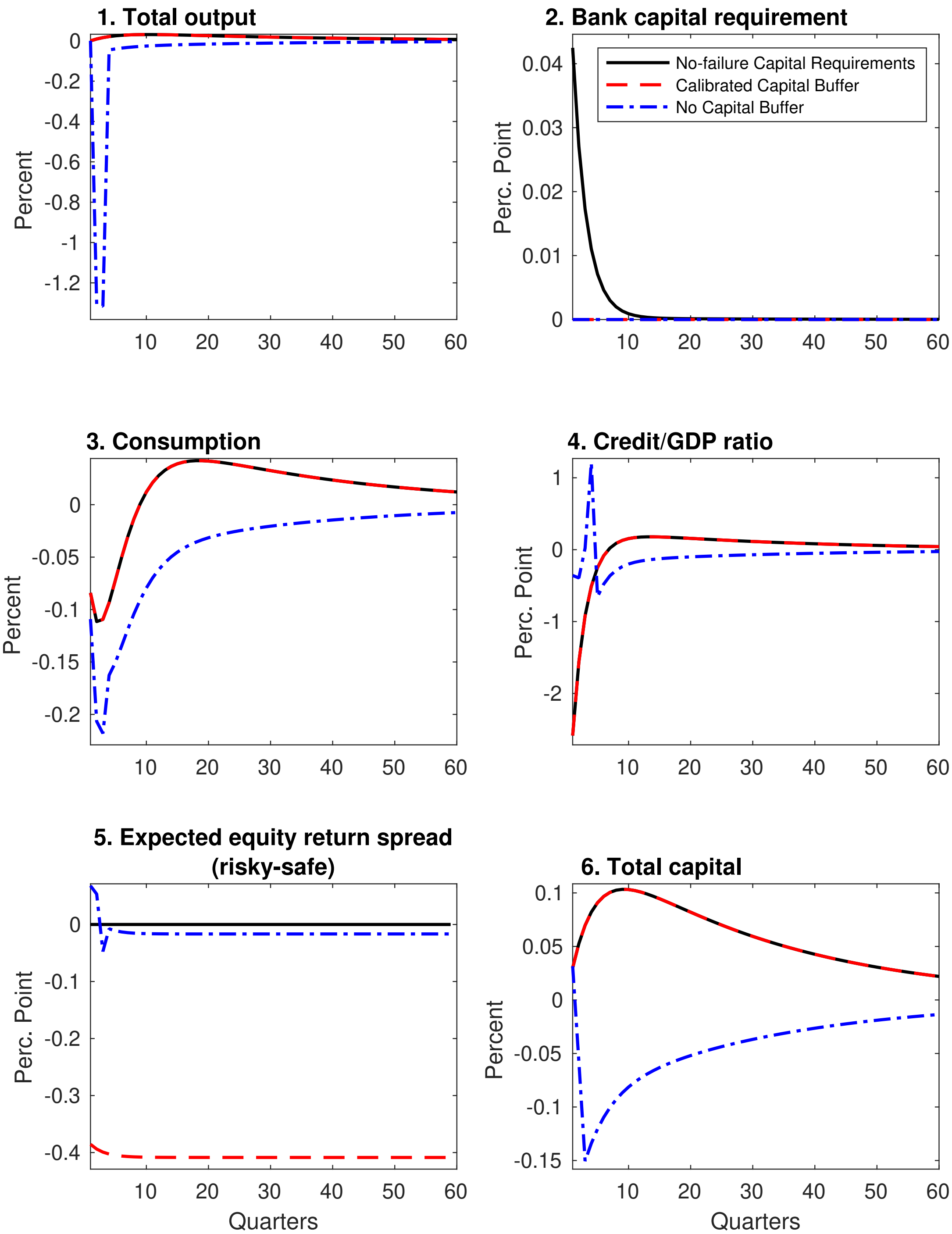
The no-failure rule raises the bank capital requirement in a recession.



Note: The solid lines, denoted as "no-failure capital requirements," show responses under a locally optimal rule that sets capital requirements to be just small enough to prevent bank failures. The dashed lines, denoted as "calibrated capital buffer," show responses under a policy rule that keeps the capital requirement at the steady-state value. The dashed-dotted lines, denoted as "no capital buffer," show responses under a capital requirement held constant at the level that prevents excessive risk-taking in the steady state.

Figure 7: **A Positive Investment Technology Shock**

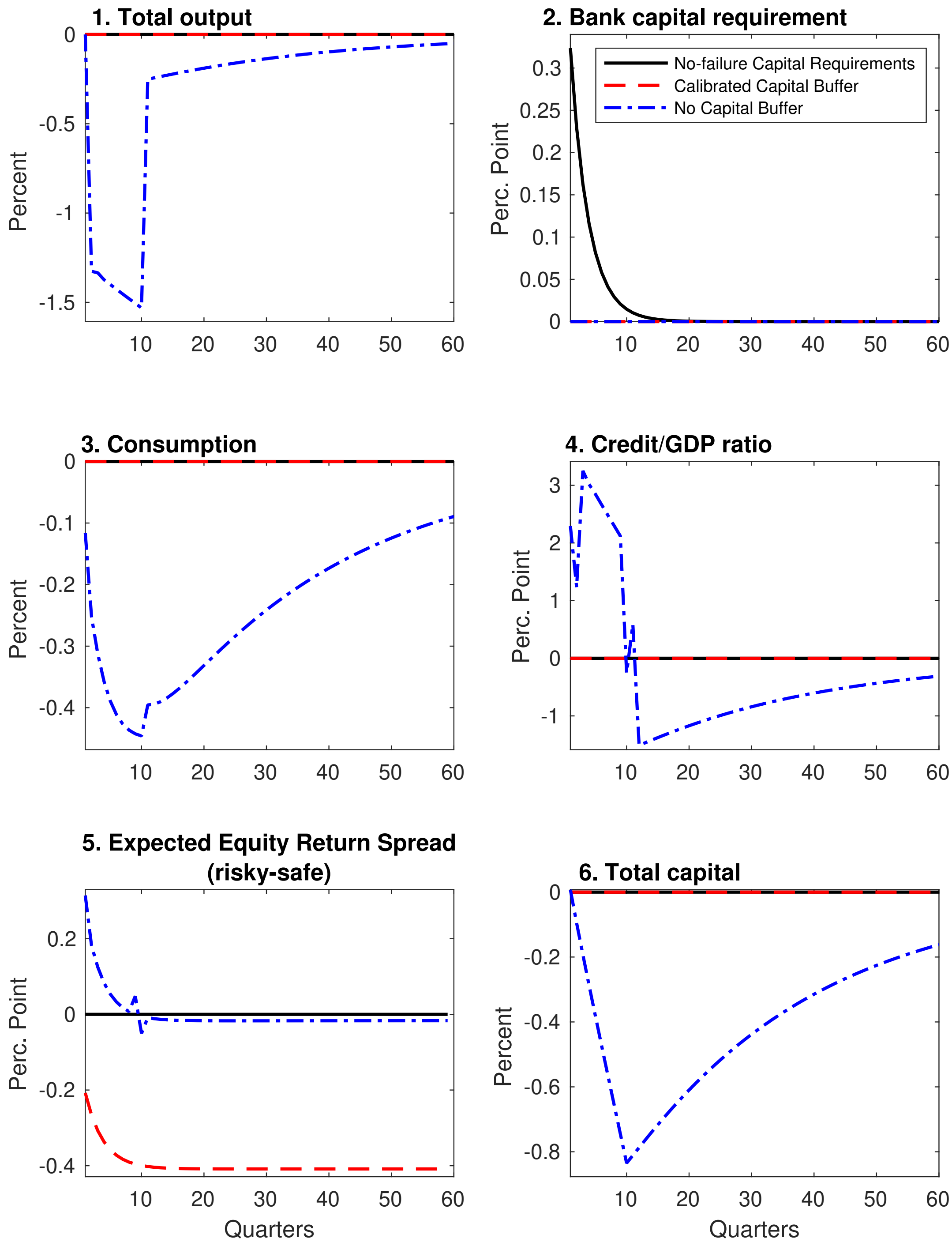
The no-failure rule raises the bank capital requirement in an expansion.



Note: The solid lines, denoted as "no-failure capital requirements," show responses under a locally optimal rule that sets capital requirements to be just small enough to prevent bank failures. The dashed lines, denoted as "calibrated capital buffer," show responses under a policy rule that keeps the capital requirement at the steady-state value. The dashed-dotted lines, denoted as "no capital buffer," show responses under a capital requirement held constant at the level that prevents excessive risk-taking in the steady state.

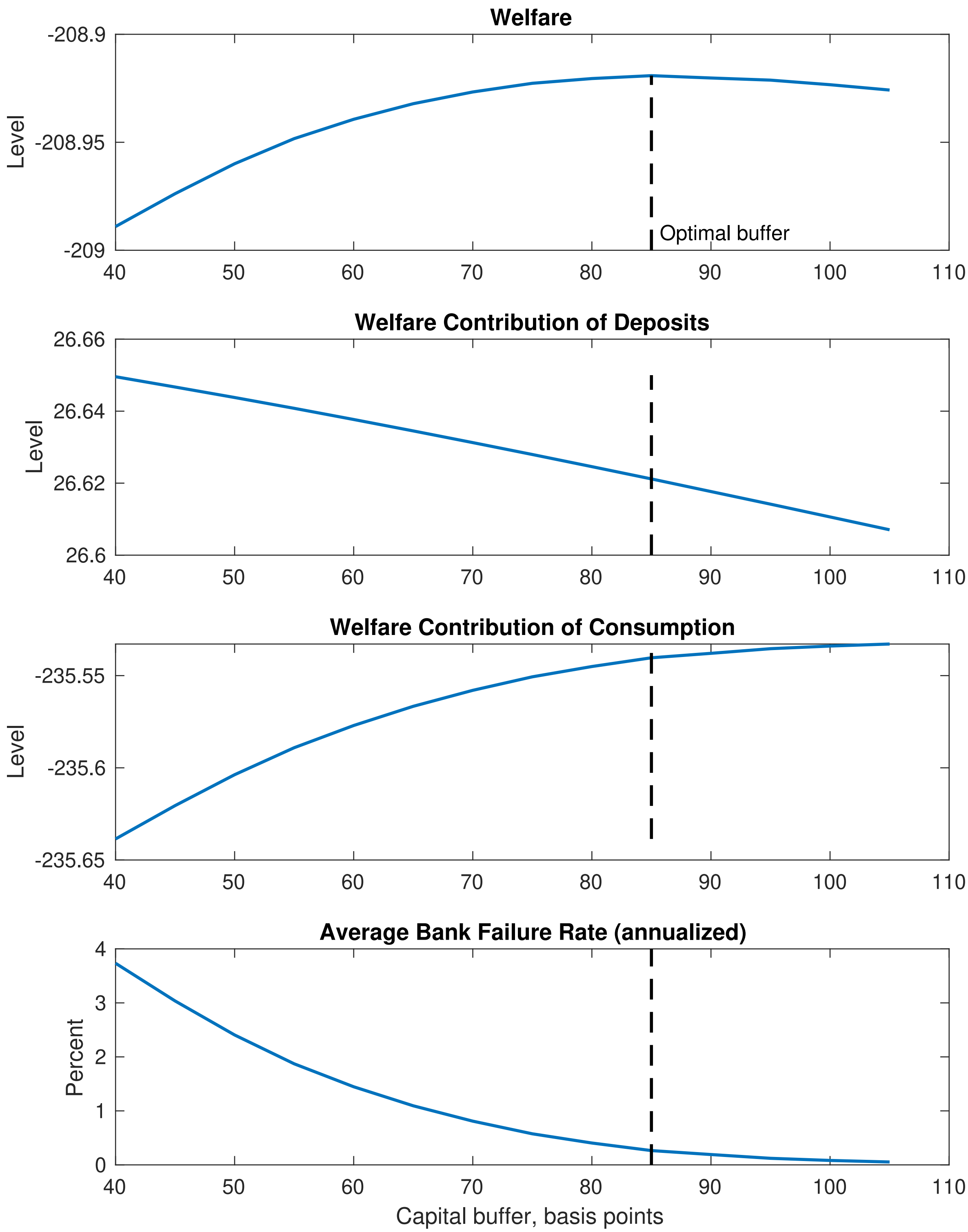
Figure 8: **A Shock that Raises the Volatility of the Returns from Risky Firms**

The no-failure rule neutralizes the macroeconomic repercussions of the shock.



Note: The solid lines, denoted as "no-failure capital requirements," show responses under a locally optimal rule that sets capital requirements to be just small enough to prevent bank failures. The dashed lines, denoted as "calibrated capital buffer," show responses under a policy rule that keeps the capital requirement at the steady-state value. The dashed-dotted lines, denoted as "no capital buffer," show responses under a capital requirement held constant at the level that prevents excessive risk-taking in the steady state.

Figure 9: **Comparing the Performance of Alternative Static Capital Buffers**



Note: The vertical dashed line denotes the welfare-maximizing static capital buffer.

Online Appendix for
A Static Capital Buffer is Hard To Beat

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December 2025

This appendix consists of four sections. Section A completes the derivation of the equilibrium conditions and provides proofs for propositions 1 and 2. Section B summarizes the equilibrium conditions for the model. Section C provides further details on the mechanisms that trigger banks to take on excessive risk. Finally, Section D derives a consumption equivalent variation that we use to interpret the welfare costs of simple rules for capital requirements.

A Derivation of Equilibrium Conditions

This section completes the derivation of the equilibrium conditions for the model. We tackle the optimization problem of households before considering banks and nonfinancial firms. We then consider the financing of the deposit insurance scheme provided by the government. We conclude by stating the resource constraints. Along the way, we prove propositions 1 and 2.

A.1 Households

In the main text, we consider a case in which we have only one representative household. The household problem involves saving through deposits and bank equity, with the capital requirement on banks determining the equilibrium split between these two assets, and providing labor services to firms inelastically. In our baseline specification, we leave the managerial labor needed to run banks unmodeled. This is not too different from abstracting from modeling the managerial services for other types of firms in the model. But there are other approaches to setting up the household problem in models that include financial intermediaries but that are in the standard RBC mold otherwise. Some of the alternatives include differentiating more explicitly between roles within households.

In this section of the appendix, we spell out how to recast the setup for the household problem described in the main text of the paper into an alternative with explicit differentiation between workers and bankers. This recasting follows the blueprint in the model of Gertler and Karadi (2011), which splits the household into workers and bankers. In that model, bankers receive an endowment untied to the utility maximization problem of the household, whereas we endogenize the endowment choice. We will show that, mutatis mutandis, there is no change in the equilibrium conditions relative to the baseline setup with a representative household.

We assume here that households are composed of workers and two bankers. The household seeks to maximize

$$\max_{C_t, D_t, E_t^s, E_t^r} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - \kappa C_{t-1})^{1-\varsigma_c} - 1}{1-\varsigma_c} + \varsigma_0 \frac{D_t^{1-\varsigma_d} - 1}{1-\varsigma_d} \right],$$

where $0 < \beta < 1$ is the worker's subjective discount factor, assumed identical across workers, i.e. $\beta_w = \beta$ for all w.

This maximization problem is subject to the sequence of budget constraints

$$\begin{aligned} C_t + D_t + E_t^s + E_t^r &= W_t + R_{t-1}^d D_{t-1} + R_t^{e,s} E_{t-1}^s + R_t^{e,r} E_{t-1}^r - T_t, \\ E_t^s &\geq 0, \\ E_t^r &\geq 0. \end{aligned}$$

Households save through deposits, D_t, which pay a gross real rate R_t^d, and by endowing bankers with initial capital: E_t^s is the endowment for bankers operating a "safe" bank, which lends to a safe firm and pays $R_{t+1}^{e,s}$ next period; E_t^r is the endowment for bankers operating a "risky" bank, which lends to a risky firm and pays $R_{t+1}^{e,r}$. As for the baseline formulation of the household problem, the returns that will be shared by the bankers are not known when the household makes the endowment decisions. By contrast, the return on deposits is known, and deposits are protected by deposit insurance. Households pay lump sum taxes, T_t, to fund the government's deposit insurance program.

Bankers run banks that they do not own.[1] They are elected to run as bankers for two periods at the end of which they return to the household. They share consumption with the household as well as the proceeds from running safe and risky banks at the end of the two-period stint as managers.

As in the new-Keynesian literature that considers sticky wages, see for example Erceg et al. (2000), we assume that households can also access a complete set of state contingent claims traded across households but in zero net supply—we leave this set unspecified in the households' objective functions and constraints. These state-contingent claims ensure equalization of consumption and deposit decisions across households, counteracting the idiosyncratic risk born by operating only one risky bank. In our baseline formulation of the household problem, we do not need this additional assumption as we think of bank equity in the same vein as a mutual fund that assigns aliquot shares of capital to different banks.

Using λ_{ct} as the Lagrangian multiplier on the budget constraint, and ζ_t^s and ζ_t^r on the non-negativity constraints for safer and risky equity, respectively, the FOCs of the representative household's problem are:

$$C_t: \quad (C_t - \kappa C_{t-1})^{-\varsigma_c} - \beta\kappa\mathbb{E}_t\left(C_{t+1} - \kappa C_t\right)^{-\varsigma_c} - \lambda_{ct} = 0, \tag{A.1}$$

$$D_t: \quad \varsigma_0 D_t^{-\varsigma_d} - \lambda_{ct} + \beta\mathbb{E}_t\left\{\lambda_{ct+1}\right\} R_t^d = 0, \tag{A.2}$$

$$E_t^s: \quad -\lambda_{ct} + \beta\mathbb{E}_t\left\{\lambda_{ct+1} R_{t+1}^{e,s}\right\} + \zeta_t^s = 0, \tag{A.3}$$

$$E_t^r: \quad -\lambda_{ct} + \beta\mathbb{E}_t\left\{\lambda_{ct+1} R_{t+1}^{e,r}\right\} + \zeta_t^r = 0 \tag{A.4}$$

together with the complementary slackness conditions:

$$\zeta_t^s E_t^s = 0, \tag{A.5}$$

$$\zeta_t^r E_t^r = 0. \tag{A.6}$$

These conditions are equivalent to the conditions that characterize the solution to the household's problem in the main text: equation (A.1) corresponds to (16) in the main text, (A.2) corresponds to (17) in the main text, (A.3) corresponds to (18) in the main text, (A.4) corresponds to (19) in the main text, (A.5) corresponds to (20) in the main text, and (A.6) corresponds to (21) in the main text.

A.2 The Bank's Problem

There is a unit measure of banks. As described in Section 2.3.2, banks maximize expected dividends

$$\max_{l_t, d_t, e_t, \sigma_t} \mathbb{E}_t\left\{\psi_{t.t+1}\left[\int_{\varepsilon_{t+1}^*}^{\infty} nw_{t+1}\, \mathrm{d}G(\varepsilon_{t+1})\right]\right\} - e_t,$$

subject to

$$l_t = e_t + d_t,$$

$$e_t \geq \gamma_t l_t,$$

$$l_t \geq 0,$$

$$\underline{\sigma} \leq \sigma_t \leq \overline{\sigma},$$

[1] Bankers can be viewed as managers with expertise in operating firms.

where $\psi_{t,t+1} = \beta \frac{\lambda_{ct+1}}{\lambda_{ct}}$ is the stochastic discount factor.

A reminder on notation is in order. We use lower-case letters to denote variables pertaining to single banks or firms, and upper-case letters to denote aggregate variables.

A.2.1 First-Order Conditions

Substituting $d_t = l_t - e_t$ into the maximization problem and writing $\mathrm{d}G(\varepsilon_{t+1})$ explicitly turn the objective into

$$\max_{l_t, e_t, \sigma_t} \mathbb{E}_t \left\{ \psi_{t,t+1} \left[\int_{\varepsilon^*_{t+1}}^{\infty} \left(\left(R^s_{t+1} + \sigma_t \frac{\varepsilon_{t+1}}{Q_t} \right) l_t - R^d_t \left(l_t - e_t \right) \right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2} \right) \mathrm{d}\varepsilon_{t+1} \right] - e_t \right\},$$

subject to

$$e_t \geq \gamma_t l_t,$$
$$l_t \geq 0,$$
$$\underline{\sigma} \leq \sigma_t \leq \overline{\sigma},$$

where $\psi_{t,t+1} = \beta \frac{\lambda_{ct+1}}{\lambda_{ct}}$ is the stochastic discount factor and $\varepsilon^*_{t+1} = \left(\frac{R^d_t - R^s_{t+1}}{\sigma_t} - \frac{R^d_t e_t}{\sigma_t l_t} \right) Q_t$ is the shield of limited liability. Note that we expressed ε^*_{t+1} from $\left(R^s_{t+1} + \sigma_t \frac{\varepsilon^*_{t+1}}{Q_t} \right) l_t - R^d_t \left(l_t - e_t \right) = 0$ to get the lower limit of the integral.

Append the Lagrangian multiplier χ_{1t} to the constraint $e_t \geq \gamma l_t$ and χ_{2t} to the constraint $l_t \geq 0$. We will split the maximization problem into two parts. We will first condition on a choice of risk, σ_t, and then show how to pin down that choice (see Section A.2.6). Accordingly, conditional on the optimal choice of σ_t, the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial l_t} = \mathbb{E}_t \left[\psi_{t,t+1} \overbrace{\left(\left(R^s_{t+1} + \sigma_t \left(\frac{R^d_t - R^s_{t+1}}{\sigma_t} - \frac{R^d_t e_t}{\sigma_t l_t} \right) \right) l_t - R^d_t \left(l_t - e_t \right) \right)}^{=0} \cdot \frac{\partial \varepsilon^*_{t+1}}{\partial l_t} \right] + \chi_{2t} +$$
$$\mathbb{E}_t \left[\int_{\varepsilon^*_{t+1}}^{\infty} \psi_{t,t+1} \frac{\partial}{\partial l_t} \left(\left(R^s_{t+1} + \sigma_t \frac{\varepsilon_{t+1}}{Q_t} \right) l_t - R^d_t \left(l_t - e_t \right) \right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2} \right) \mathrm{d}\varepsilon_{t+1} \right] - \gamma \chi_{1t} = 0.$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = -\mathbb{E}_t \left[\psi_{t,t+1} \overbrace{\left(\left(R^s_{t+1} + \sigma_t \left(\frac{R^d_t - R^s_{t+1}}{\sigma_t} - \frac{R^d_t e_t}{\sigma_t l_t} \right) \right) l_t - R^d_t \left(l_t - e_t \right) \right)}^{=0} \cdot \frac{\partial \varepsilon^*_{t+1}}{\partial e_t} \right] + \chi_{1t} +$$
$$\mathbb{E}_t \left[\int_{\varepsilon^*_{t+1}}^{\infty} \psi_{t,t+1} \frac{\partial}{\partial e_t} \left(\left(R^s_{t+1} + \sigma_t \frac{\varepsilon_{t+1}}{Q_t} \right) l_t - R^d_t \left(l_t - e_t \right) \right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2} \right) \mathrm{d}\varepsilon_{t+1} \right] - 1 = 0,$$

together with the complementary slackness conditions:

$$\begin{aligned}
&\chi_{1t}\left(e_t-\gamma_t l_t\right)=0,\\
&\chi_{2t} l_t=0,\\
&e_t-\gamma_t l_t\geq 0,\\
&l_t\geq 0,\\
&\chi_{1t}\geq 0,\\
&\chi_{2t}\geq 0,
\end{aligned}$$

We are using the Leibniz integral rule above to find the partial derivatives of the profit function. Note that the first term is zero in the differentiation because the upper limit of the integral does not depend on any of the choice variables.

Next, express the integrals in the first-order conditions above using the erf function, wherever possible. Note that we omit the stochastic discount factor and the expectation operator in writing up the expressions of the next integrals. We include those terms in the final exposition.

Work on $\frac{\partial}{\partial l_t}$:

$$\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty} \frac{\partial}{\partial l_t}\left(\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}\right)l_t-R_t^d\left(l_t-e_t\right)\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}=$$

$$\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}-R_t^d\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}=$$

Break the calculation of the integral into two parts.

$$\frac{\sigma_t}{Q_t}\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}\varepsilon_{t+1}\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}+$$

$$\left(R_{t+1}^s-R_t^d\right)\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}.$$

Work on the first part

$$\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}\varepsilon_{t+1}\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1},$$

by introducing a change in variables to recast the integral in terms of the Standard Normal distribution. Use $v=\frac{\varepsilon_{t+1}+\xi}{\sqrt{2}\tau_t}$, or equivalently $\varepsilon_{t+1}=v\sqrt{2}\tau_t-\xi$, and remember that for the change $x=\varphi(t)$, the integral $\int_{\varphi(a)}^{\varphi(b)} f(x)dx$ becomes $\int_a^b f(\varphi(t))\varphi'(t)dt$. Here, we use that $dv = \frac{d\varepsilon_{t+1}}{\sqrt{2}\tau_t}$, so we need to multiply dv by $\sqrt{2}\tau_t$ to express $d\varepsilon_{t+1}$ in terms of dv. Moreover, we need to transform the lower limit using v. So we need to add ξ to the lower limit of the integral and divide the result by $\sqrt{2}\tau_t$.

$$\int\limits_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty} \left(v\sqrt{2}\tau_t-\xi\right)\frac{\sqrt{2}\tau_t}{\sqrt{2\pi\tau_t^2}}\exp\left(-v^2\right)\,\mathrm{d}v =$$

$$\frac{\sqrt{2}\tau_t}{\sqrt{\pi}}\int\limits_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty} v\exp\left(-v^2\right)\,\mathrm{d}v-\frac{\xi}{\sqrt{\pi}}\int\limits_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty}\exp\left(-v^2\right)\,\mathrm{d}v =$$

$$-\frac{\sqrt{2}\tau_t}{2\sqrt{\pi}}\exp\left(-v^2\right)\Bigg|_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty}-\frac{\xi}{\sqrt{\pi}}\left[\int\limits_0^{\infty}\exp\left(-v^2\right)\,\mathrm{d}v-\int\limits_0^{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}\exp\left(-v^2\right)\,\mathrm{d}v\right] =$$

$$0+l_t\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)^2\right)-$$

$$\frac{\xi}{\sqrt{\pi}}\left[\frac{\sqrt{\pi}}{2}\mathrm{erf}(\infty)-\frac{\sqrt{\pi}}{2}\mathrm{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right] =$$

$$\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)^2\right)-\frac{\xi}{2}\left[1-\mathrm{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right],$$

where we used that $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp\left(-v^2\right)$.

Therefore,

$$\begin{aligned}&\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}\varepsilon_{t+1}\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1} =\\&\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)^2\right)-\frac{\xi}{2}\left[1-\mathrm{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right]\end{aligned} \tag{A.7}$$

Moving to the second part, let us express $\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}\left(\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\right)\mathrm{d}\varepsilon_{t+1}$ in terms of the error function. Again, use the transformation $v = \frac{\varepsilon_{t+1}+\xi}{\sqrt{2}\tau_t}$ or $\varepsilon_{t+1} = v\sqrt{2}\tau_t - \xi$

$$\int_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty} \frac{\sqrt{2}\tau_t}{\sqrt{2\pi\tau_t^2}} \exp\left(-v^2\right) \mathrm{d}v = \frac{1}{\sqrt{\pi}} \int_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty} \exp\left(-v^2\right) \mathrm{d}v = \frac{1}{2}\left(1-\operatorname{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right).$$

Therefore,

$$\int_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty} \left(\frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\right) \mathrm{d}\varepsilon_{t+1} = \frac{1}{2}\left(1-\operatorname{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right). \tag{A.8}$$

Combining the results for the two parts and simplifying,

$$\mathbb{E}_t\left[\int_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty} \frac{\partial}{\partial l_t}\left(\left(\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}\right)l_t - R_t^d\left(l_t-e_t\right)\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}\right] =$$

$$\mathbb{E}_t\left[\int_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty} \left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}-R_t^d\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}\right] =$$

$$\mathbb{E}_t\left[\frac{\sigma_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)^2\right) - \frac{\sigma_t\xi}{2Q_t}\left[1-\operatorname{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right]\right] +$$

$$\mathbb{E}_t\left[\left(R_{t+1}^s-R_t^d\right)\frac{1}{2}\left(1-\operatorname{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right)\right] =$$

$$\mathbb{E}_t\left[\frac{\sigma_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)^2\right) + \left(\frac{R_{t+1}^s-\frac{\sigma_t\xi}{Q_t}-R_t^d}{2}\right)\left[1-\operatorname{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right]\right].$$

Similarly, work on $\frac{\partial}{\partial e_t}$

$$\int_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_{t+1}^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty} \frac{\partial}{\partial e_t}\left(\left(\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}\right)l_t - R_t^d\left(l_t-e_t\right)\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1} =$$

$$\int_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_{t+1}^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty} R_t^d\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1} = R_t^d\frac{1}{2}\left(1-\operatorname{erf}\left(\frac{R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right).$$

In sum, the FOCs can be written as follows:

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\frac{\sigma_t}{Q_t} \frac{\tau_t}{\sqrt{2\pi}} \exp\left(- \left(\frac{\left(R^d_t \left(1 - \frac{e_t}{l_t}\right) - R^s_{t+1}\right) Q_t + \xi\sigma_t}{\sigma_t \sqrt{2}\tau_t} \right)^2 \right) + \left(\frac{R^s_{t+1} - \frac{\sigma_t \xi}{Q_t} - R^d_t}{2} \right) \left[1 - \text{erf}\left(\frac{\left(R^d_t \left(1 - \frac{e_t}{l_t}\right) - R^s_{t+1}\right) Q_t + \xi\sigma_t}{\sigma_t \sqrt{2}\tau_t} \right) \right] \right] \right\} + \chi_{2t} = \gamma\chi_{1t}, \tag{A.9}$$

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R^d_t \frac{1}{2} \left(1 - \text{erf}\left(\frac{\left(R^d_t \left(1 - \frac{e_t}{l_t}\right) - R^s_{t+1}\right) Q_t + \xi\sigma_t}{\sigma_t \sqrt{2}\tau_t} \right) \right) \right] \right\} - 1 + \chi_{1t} = 0. \tag{A.10}$$

Finally, the complementary slackness conditions can be expressed as

$$(e_t - \gamma l_t)\, \chi_{1t} = 0, \tag{A.11}$$

$$l_t \chi_{2t} = 0. \tag{A.12}$$

A.2.2 Proof of Proposition 1

In equilibrium, capital requirements always bind; that is,

$$e_t = \gamma_t l_t.$$

We will show that the Lagrange multiplier associated with this constraint is always positive, hence the constraint must bind. To this end, we also employ the household problem's first-order conditions (FOCs) with respect to equity, described in Appendix A.1.

Equations (A.3) and (A.4) can be expressed as

$$\beta \mathbb{E}_t \left\{ \frac{\lambda_{ct+1}}{\lambda_{ct}} R^{e,i}_{t+1} \right\} = 1 - \frac{\zeta^i_t}{\lambda_{ct}},$$

where $i \in \{s, r\}$ denotes the type of equity. In this expression, substitute equation (A.10) for 1. Therefore,

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R^d_t \frac{1}{2} \left(1 - \text{erf}\left(\frac{\left(R^d_t \left(1 - \frac{e^i_t}{l^i_t}\right) - R^s_{t+1}\right) Q_t + \xi\sigma^i_t}{\sigma^i_t \sqrt{2}\tau_t} \right) \right) \right] - R^{e,i}_{t+1} \right\} - \frac{\zeta^i_t}{\lambda_{ct}} + \chi^i_{1t} = 0. \tag{A.13}$$

Since the range of the erf function is between -1 and 1, i.e.$-1 \leq \text{erf}(x) \leq 1$, we know that the following expression is between Ψ^{i*}_1 and Ψ^{i*}_2:

$$\Psi^{i*}_1 \leq \mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R^d_t \frac{1}{2} \left(1 - \text{erf}\left(\frac{\left(R^d_t \left(1 - \frac{e^i_t}{l^i_t}\right) - R^s_{t+1}\right) Q_t + \xi\sigma^i_t}{\sigma^i_t \sqrt{2}\tau_t} \right) \right) - R^{e,i}_{t+1} \right] \right\} \leq \Psi^{i*}_2,$$

where

$$\Psi_1^{i*} = \mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[0 - R_{t+1}^{e,i} \right] \right\},$$
$$\Psi_2^{i*} = \mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R_t^d - R_{t+1}^{e,i} \right] \right\}.$$

Plugging λ_{ct} from equations (A.3) an (A.4) into equation (A.2), we get:

$$\mathbb{E}_t \left\{ \beta \lambda_{ct+1} \left[R_t^d - R_{t+1}^{e,i} \right] \right\} = -\varsigma_0 D_t^{-\varsigma_d} + \zeta_t^i.$$

Note that $\varsigma_0 D_t^{-\varsigma_d} > 0$ under the usual (and mild) assumptions on the preferences for liquidity. Moreover, the Lagrangian multiplier on the households budget constraint, λ_{ct}, is positive. It reflects the fact that the budget constraint always binds given the standard assumptions on the preferences (the Inada conditions). The latest expression is transformed into the following after dividing it by λ_{ct}:

$$\underbrace{\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R_t^d - R_{t+1}^{e,i} \right] \right\}}_{=\Psi_2^{i*}} - \frac{\zeta_t^i}{\lambda_{ct}} = -\frac{\varsigma_0 D_t^{-\varsigma_d}}{\lambda_{ct}} < 0.$$

Thus, $\Psi_2^{i*} < \frac{\zeta_t^i}{\lambda_{ct}}$.

Rewriting Equation (A.13)

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R_t^d \frac{1}{2} \left(1 - \operatorname{erf} \left(\frac{\left(R_t^d \left(1 - \frac{e_t^i}{l_t^i} \right) - R_{t+1}^s \right) Q_t + \xi \sigma_t^i}{\sigma_t^i \sqrt{2} \tau_t} \right) \right) \right] - R_{t+1}^{e,i} \right\} = \frac{\zeta_t^i}{\lambda_{ct}} - \chi_{1t}^i,$$

combine it with $\Psi_2^{i*} < \frac{\zeta_t^i}{\lambda_{ct}}$ to find

$$\frac{\zeta_t^i}{\lambda_{ct}} - \chi_{1t} < \Psi_2^{i*} < \frac{\zeta_t^i}{\lambda_{ct}}.$$

Hence, $\chi_{1t}^i > 0$ for each $i \in \{s, r\}$. $\square$

A.2.3 Combined First-Order Conditions

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\frac{\sigma_t}{Q_t} \frac{\tau_t}{\sqrt{2\pi}} \exp \left(- \left(\frac{\left(R_t^d \left(1 - \frac{e_t}{l_t} \right) - R_{t+1}^s \right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right)^2 \right) + \right.\right.$$
$$\left.\left. \left(\frac{R_{t+1}^s - \frac{\sigma_t \xi}{Q_t} - R_t^d}{2} \right) \left[1 - \operatorname{erf} \left(\frac{\left(R_t^d \left(1 - \frac{e_t}{l_t} \right) - R_{t+1}^s \right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right) \right] \right] \right\} + \chi_{2t} = \gamma \chi_{1t},$$
$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R_t^d \frac{1}{2} \left(1 - \operatorname{erf} \left(\frac{\left(R_t^d \left(1 - \frac{e_t}{l_t} \right) - R_{t+1}^s \right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right) \right) \right] \right\} - 1 + \chi_{1t} = 0.$$

Since $\chi_{1t} > 0$, multiply the second equation by γ_t and add it to the first equation using $\frac{e_t}{l_t} = \gamma_t$. Therefore, the FOCs can be combined into:

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\frac{\sigma_t}{Q_t} \frac{\tau_t}{\sqrt{2\pi}} \exp \left(- \left(\frac{\left(R_t^d\left(1-\gamma_t\right) - R_{t+1}^s\right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right)^2 \right) + \frac{1}{2} \left(R_{t+1}^s - \frac{\sigma_t \xi}{Q_t} - R_t^d (1-\gamma_t) \right) \left[1 - \operatorname{erf} \left(\frac{\left(R_t^d\left(1-\gamma_t\right) - R_{t+1}^s\right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right) \right] \right] \right\} = \gamma_t - \chi_{2t}, \tag{A.14}$$

$$\chi_{2t} l_t = 0. \tag{A.15}$$

A.2.4 Zero-Profit Condition

In step 1, we derive the expression of the zero-profit condition under all states of nature. In step 2, we show that this zero-profit condition implies the FOCs derived in Appendix A.2.3.

Step 1: Since there is no agency problem between banks and households, this condition captures the fact that all the profits (or losses) are distributed to equity holders after realization of shocks at the beginning of each period. In each aggregate state, banks whose investments in risky firms pan out will have returns that satisfy on average (over the realizations of the idiosyncratic shock) $\left[\left(R_{t+1}^s + \frac{\sigma_t}{Q_t}\right) l_t - R_t^d \left(l_t - e_t\right)\right] - \int R_{t+1,b}^e(b) \cdot e_t = 0$, where the bounds of the integral are chosen such that we integrate over banks for which the profit is non-negative, while banks whose risky investments earn low (negative) returns will have $R_{t+1,b}^e = 0$. Therefore,

$$R_{t+1}^e = \int_{\left(\frac{R_t^d(1-\gamma_t)-R_{t+1}^s}{\sigma_t}\right)Q_t}^{\infty} \frac{\left(\left(R_{t+1}^s + \sigma_t \frac{\varepsilon_{t+1}}{Q_t}\right) l_t - R_t^d d_t\right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1}}{e_t} + \int_{-\infty}^{\left(\frac{R_t^d(1-\gamma_t)-R_{t+1}^s}{\sigma_t}\right)Q_t} 0 \cdot \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1} =$$

$$\frac{1}{e_t} \int_{\left(\frac{R_t^d(1-\gamma_t)-R_{t+1}^s}{\sigma_t}\right)Q_t}^{\infty} \left(R_{t+1}^s l_t - R_t^d d_t\right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1} + \frac{1}{e_t} \int_{\left(\frac{R_t^d(1-\gamma_t)-R_{t+1}^s}{\sigma_t}\right)Q_t}^{\infty} \sigma_t \frac{\varepsilon_{t+1}}{Q_t} l_t \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1} =$$

$$\frac{1}{e_t} \left[\left(R_{t+1}^s l_t - R_t^d d_t\right) \frac{1}{2} \left(1 - \operatorname{erf}\left(\frac{\left(R_t^d(1-\gamma_t) - R_{t+1}^s\right) Q_t + \xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right) + \frac{\sigma_t l_t}{Q_t} \left(\frac{\tau_t}{\sqrt{2\pi}} \exp\left(-\left(\frac{\left(R_t^d(1-\gamma_t)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)^2\right) - \frac{\xi}{2}\left[1 - \operatorname{erf}\left(\frac{\left(R_t^d(1-\gamma_t)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right]\right)\right] =$$

$$\frac{l_t}{e_t}\left\{\frac{\sigma_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d(1-\gamma_t)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)^2\right)+\right.$$
$$\left.\frac{1}{2}\left(R_{t+1}^s-\frac{\sigma_t\xi}{Q_t}-R_t^d\left(1-\gamma_t\right)\right)\left[1-\text{erf}\left(\frac{\left(R_t^d(1-\gamma_t)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right]\right\}.$$

Since $\frac{l_t}{e_t}=\frac{1}{\gamma_t}$, we can rewrite the latter condition as (using that it holds for each $i\in\{s,r\}$):

$$\begin{aligned}R_{t+1}^{e,i}=&\frac{1}{\gamma_t}\frac{\sigma_t^i}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)^2\right)+\\&\frac{1}{\gamma_t}\frac{1}{2}\left(R_{t+1}^s-\frac{\sigma_t^i\xi}{Q_t}-R_t^d\left(1-\gamma_t\right)\right)\left[1-\text{erf}\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)\right].\end{aligned}\tag{A.16}$$

Step 2: Note that the combined FOC from Appendix A.2.3 can be expressed as:

$$\begin{aligned}&\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{\sigma_t^i}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)^2\right)+\right.\right.\\&\left.\left.\frac{1}{2}\left(R_{t+1}^s-\frac{\sigma_t^i\xi}{Q_t}-R_t^d(1-\gamma_t)\right)\left[1-\text{erf}\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)\right]\right]\right\}=\\&\gamma_t-\chi_{2t}^i=\gamma_t\left(\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}^{e,i}\right\}+\frac{\zeta_t^i}{\lambda_{ct}}\right)-\chi_{2t}^i,\end{aligned}\tag{A.17}$$

where we substitute for 1 from Household's FOC with respect to the two types of equity, i.e.,

$$\beta\mathbb{E}_t\left\{\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}^{e,i}\right\}=1-\frac{\zeta_t^i}{\lambda_{ct}}$$

for each $i\in\{s,r\}$.

When $l_t^i>0$, the complementary slackness conditions in equations (A.5), (A.6), and (A.15) imply both $\zeta_t^i=0$ and $\chi_{2t}^i=0$. Plugging these results into equation (A.17):

$$\begin{aligned}\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}^{e,i}\right\}=&\frac{1}{\gamma_t}\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{\sigma_t^i}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)^2\right)+\right.\right.\\&\left.\left.\frac{1}{2}\left(R_{t+1}^s-\frac{\sigma_t^i\xi}{Q_t}-R_t^d(1-\gamma_t)\right)\left[1-\text{erf}\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)\right]\right]\right\}.\end{aligned}\tag{A.18}$$

Notice that equation (A.18) differs from the zero-profit condition in equation (A.18) by only the stochastic discount factor attached to the both sides of the equation. Therefore, the zero-profit condition implies the FOC.

A.2.5 Expression of Expected Dividends

Expected dividends (valued on date t) are defined as

$$\Omega\left(\sigma_t;\, l_t,\, d_t,\, e_t\right) = -e_t +$$
$$\mathbb{E}_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\int\limits_{\left(\frac{R_t^d(l_t-e_t)}{\sigma_t l_t}-\frac{R_{t+1}^s}{\sigma_t}\right)Q_t}^{\infty}\left(\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}\right)l_t - R_t^d\left(l_t-e_t\right)\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}\right]$$

In Appendix A.2.1, we calculated all the necessary ingredients for the above integral. Multiply equation (A.7) by $\frac{\sigma_t l_t}{Q_t}$ and make a sum with equation (A.8) multiplied by $R_{t+1}^s l_t - R_t^d(l_t-e_t))$. Use that the capital requirement always binds, i.e., $e_t = \gamma_t l_t$ Therefore,

$$\begin{aligned}&\Omega\left(\sigma_t;\, l_t,\, d_t,\, e_t\right) = -l_t\gamma_t +\\ &l_t\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{\sigma_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)^2\right)+\right.\right.\\ &\left.\left.\frac{\left(R_{t+1}^s-R_t^d\left(1-\gamma_t\right)-\frac{\sigma_t\xi}{Q_t}\right)}{2}\left[1-\mathrm{erf}\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right]\right]\right\}.\end{aligned} \tag{A.19}$$

We define ε_{t+1}^* as the realization of the idiosyncratic shock below which the bank's net worth is negative; that is, $\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}^*}{Q_t}\right)l_t - R_t^d d_t = 0$. Expressing

$$\varepsilon_{t+1}^* = -\frac{Q_t}{\sigma_t}\left[R_{t+1}^s - R_t^d\left(1-\gamma_t\right)\right]$$

and plugging it into the cdf of the Normal distribution:

$$G(\varepsilon_{t+1}^*) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{\varepsilon_{t+1}^*+\xi}{\tau_t\sqrt{2}}\right)\right] = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{-\frac{Q_t}{\sigma_t}\left[R_{t+1}^s-R_t^d\left(1-\gamma_t\right)\right]+\xi}{\tau_t\sqrt{2}}\right)\right].$$

Therefore, $1-G(\varepsilon_{t+1}^*) =$

$$1-\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{-\frac{Q_t}{\sigma_t}\left[R_{t+1}^s-R_t^d\left(1-\gamma_t\right)\right]+\xi}{\tau_t\sqrt{2}}\right)\right] = \frac{1}{2}-\frac{1}{2}\mathrm{erf}\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right).$$

Using these results, we can re-write equation (A.19) as follows:

$$\begin{aligned}\Omega\left(\sigma_t;\, l_t,\, d_t,\, e_t\right) = -l_t\gamma_t + l_t\mathbb{E}_t&\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\left(R_{t+1}^s-R_t^d\left(1-\gamma_t\right)-\frac{\xi\sigma_t}{Q_t}\right)\left(1-G(\varepsilon_{t+1}^*)\right)+\right.\right.\\ &\left.\left.\left(\frac{\sigma_t}{Q_t}\right)\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\varepsilon_{t+1}^*+\xi}{\tau_t\sqrt{2}}\right)^2\right)\right]\right\}.\end{aligned} \tag{A.20}$$

By denoting

$$\omega_1 \equiv \left(R^s_{t+1} - R^d_t\left(1-\gamma_t\right) - \frac{\xi\sigma_t}{Q_t}\right)\left(1 - G(\varepsilon^*_{t+1})\right),$$
$$\omega_2 \equiv \left(\frac{\sigma_t}{Q_t}\right)\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\varepsilon^*_{t+1}+\xi}{\tau_t\sqrt{2}}\right)^2\right),$$

Equation (A.20) can be described as

$$\Omega(\sigma_t;\, l_t,\, d_t,\, e_t) = \left(\mathbb{E}_t\left[\psi_{t,t+1}\left(\omega_1+\omega_2\right)\right] - \gamma_t\right) l_t.$$

A.2.6 Choice of Risk

In this section of the appendix, we prove that the expected dividend function of banks is convex in the risk parameter σ_t. This result guarantees that banks choose either the maximum risk, $\overline{\sigma}$, or the minimum risk, $\underline{\sigma}$, to maximize expected shareholder value, net of initial equity investment. So all the intermediate values of σ_t, which may result from the first-order conditions with respect to σ_t, are not optimal.

The proof provided here generalizes the proof of Van den Heuvel (2008) to the presence of aggregate uncertainty. Our proof applies to an arbitrary distribution of the idiosyncratic shock, ε_{t+1}, with non-positive mean, so our example of a Normal distribution considered in the analysis is not a special case that can drive our results.

Assumption. *ε has a cumulative distribution function G_ε with support $[\underline{\varepsilon},\, \overline{\varepsilon}]$, with $\underline{\varepsilon} < 0 < \overline{\varepsilon}$. The mean of ε is equal to $-\xi$ ($\xi > 0$). ε is independent of the aggregate shock. The aggregate shock does not depend on the choice of σ_t.*

Note that we do not restrict the analysis to the bounded support[2], so $\underline{\varepsilon}$ and $\overline{\varepsilon}$ can take $-\infty$ and $+\infty$, respectively. Note that G_ε need not be continuous.

Let $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) \equiv \left(\frac{R^d_t d_t}{\sigma_t l_t} - \frac{R^s_{t+1}}{\sigma_t}\right) Q_t = \frac{R^d_t(1-\gamma_t)-R^s_{t+1}}{\sigma_t} Q_t$, where the latter equation uses the result that the capital requirement constraint always binds. It denotes the realization of the idiosyncratic shock below which the bank's net worth is negative. Let $\pi(\sigma_t,\, R^s_{t+1}) = E_\varepsilon\left[\left(\left(R^s_{t+1} + \frac{\sigma_t\varepsilon}{Q_t}\right) l_t - R^d_t d_t\right)^+\right]$ be a function of expected net worth (taken over the idiosyncratic shock only) under some realization of R^s_{t+1} which is considered to be fixed in this function. To account for the aggregate uncertainty, R^s_{t+1} needs to be a random variable. Therefore, expected net worth taken into account both idiosyncratic and aggregate uncertainty is

$$\Pi(\sigma_t) = \int_\Omega \psi_{t,t+1}\pi\left(\sigma_t,\, R^s_{t+1}(\omega)\right) P(d\omega) = \mathbb{E}_t\left[\psi_{t,t+1}\int_{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})}^{\overline{\varepsilon}}\left(\left(R^s_{t+1} + \frac{\sigma_t\varepsilon}{Q_t}\right) l_t - R^d_t d_t\right) dG_\varepsilon\right] =$$
$$\mathbb{E}_t\left[\psi_{t,t+1}\int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\left(\left(R^s_{t+1} + \frac{\sigma_t\varepsilon}{Q_t}\right) l_t - R^d_t d_t\right) dG_\varepsilon\right] - \mathbb{E}_t\left[\psi_{t,t+1}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})}\left(\left(R^s_{t+1} + \frac{\sigma_t\varepsilon}{Q_t}\right) l_t - R^d_t d_t\right) dG_\varepsilon\right] =$$

[2] Unbounded support is more relevant if we consider aggregate risk

$$\mathbb{E}_t\left[\psi_{t,t+1}\left(R^s_{t+1}l_t - R^d_t d_t - \frac{\sigma_t\xi}{Q_t}l_t\right)\right] - \frac{\sigma_t l_t}{Q_t}\mathbb{E}_t\left[\psi_{t,t+1}\int\limits_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})}\left(\varepsilon - \hat{\varepsilon}(\sigma_t,\, R^s_{t+1})\right)dG_\varepsilon\right] =$$

$$\mathbb{E}_t\left[\psi_{t,t+1}\left(R^s_{t+1}l_t - R^d_t d_t\right)\right] + \frac{l_t}{Q_t}\left(\sigma_t\mathbb{E}_t\left[\psi_{t,t+1}\int\limits_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})}\left(\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) - \varepsilon\right)dG_\varepsilon\right] - \sigma_t\xi\mathbb{E}_t\psi_{t,t+1}\right).$$

Note that in the derivations above we express $\left(R^s_{t+1} + \frac{\sigma_t\varepsilon}{Q_t}\right)l_t - R^d_t d_t$ in terms of $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})$ and ε using the definition of $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})$.

The proof below shows that $\Pi(\sigma_t)$ is convex in σ_t. Note that the expression of $\Pi(\sigma_t)$ involves the term which is linear in σ_t and $\frac{l_t}{Q_t} \geq 0$. Moreover, $\psi_{t,t+1} > 0$. Therefore, the sufficient condition for $\Pi(\sigma_t)$ to be convex in σ_t is that

$$H(\sigma_t) \equiv \mathbb{E}_t\left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t)}\left(\hat{\varepsilon}(\sigma_t) - \varepsilon\right)dG_\varepsilon\right]\sigma_t$$

is convex in σ_t.

Claim. $H(\sigma_t) \equiv \mathbb{E}_t\left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t)}\left(\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) - \varepsilon\right)dG_\varepsilon\right]\sigma_t$ *is convex in* σ_t*:*

Proof. Steps of the proof: □

1. Define $h(\sigma_t,\, R^s_{t+1}) \equiv \sigma_t\left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})}\left(\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) - \varepsilon\right)dG_\varepsilon\right]$ in which the aggregate uncertainty is taken off. Consider 3 cases:

   (a) The realization of R^s_{t+1} is such that $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) = \frac{R^d_t(1-\gamma_t) - R^s_{t+1}}{\sigma_t} > 0$, so $R^s_{t+1} < R^d_t\left(1-\gamma_t\right)$,

   (b) The realization of R^s_{t+1} is such that $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) = \frac{R^d_t(1-\gamma_t) - R^s_{t+1}}{\sigma_t} < 0$, so $R^s_{t+1} > R^d_t\left(1-\gamma_t\right)$,

   (c) The realization of R^s_{t+1} is such that $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) = \frac{R^d_t(1-\gamma_t) - R^s_{t+1}}{\sigma_t} = 0$, so $R^s_{t+1} = R^d_t\left(1-\gamma_t\right)$,

   Show that $h(\sigma_t,\, R^s_{t+1})$ is convex in σ_t in cases 1a and 1b and $h(\sigma_t,\, R^s_{t+1})$ is linear in σ_t in case 1c.

2. Employ the argument that convexity is preserved under non-negative scaling and addition (guaranteed by the expectation operator over the aggregate uncertainty) to find that $H(\sigma_t)$ is convex.

Let's show each step of the proof formally:

1. Let $\sigma_{1t} < \sigma_{2t}$ and, for $\lambda \in (0,\, 1)$, define $\sigma_{\lambda t} = \lambda\sigma_{1t} + (1-\lambda)\sigma_{2t}$. Let $\hat{\varepsilon}_i = \hat{\varepsilon}(\sigma_{it},\, R^s_{t+1}) \equiv \frac{R^d_t(1-\gamma_t) - R^s_{t+1}}{\sigma_{it}}Q_t$, for $i = 1,\, 2,\, \lambda$. Therefore,

$$\hat{\varepsilon}_1\sigma_{1t} = \hat{\varepsilon}_2\sigma_{2t} = \hat{\varepsilon}_\lambda(\lambda\sigma_{1t} + (1-\lambda)\sigma_{2t}) \tag{A.21}$$

   Using the definition of the convex function, we need to show that

$$h(\sigma_{\lambda t}) \leq \lambda h(\sigma_{1t}) + (1-\lambda)h(\sigma_{2t}).$$

   (a) $R^s_{t+1} < R^d_t\left(1-\gamma_t\right)$: it implies that $\hat{\varepsilon}_2 < \hat{\varepsilon}_\lambda < \hat{\varepsilon}_1$,

$$h(\sigma_{\lambda t}) = (\lambda\sigma_{1t} + (1-\lambda)\sigma_{2t})\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_{\lambda t})}\left(\hat{\varepsilon}(\sigma_{\lambda t}) - \varepsilon\right)dG_\varepsilon\right\} =$$

$$\lambda\sigma_{1t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon-\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon\right\}+$$
$$(1-\lambda)\sigma_{2t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon+\int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon\right\}=$$
$$\lambda\sigma_{1t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_1-\varepsilon)\,dG_\varepsilon+(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_1)\,G_\varepsilon(\hat{\varepsilon}_1)+\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}(\varepsilon-\hat{\varepsilon}_\lambda)\,dG_\varepsilon\right\}+$$
$$(1-\lambda)\sigma_{2t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}(\hat{\varepsilon}_2-\varepsilon)\,dG_\varepsilon+(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_2)\,G_\varepsilon(\hat{\varepsilon}_2)+\int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon\right\}\leq$$
$$\lambda\sigma_{1t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_1-\varepsilon)\,dG_\varepsilon+(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_1)\,G_\varepsilon(\hat{\varepsilon}_1)+\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_1-\hat{\varepsilon}_\lambda)\,dG_\varepsilon\right\}+$$
$$(1-\lambda)\sigma_{2t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}(\hat{\varepsilon}_2-\varepsilon)\,dG_\varepsilon+(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_2)\,G_\varepsilon(\hat{\varepsilon}_2)+\int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_2)\,dG_\varepsilon\right\},$$

where the inequality sign comes from $\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}(\varepsilon-\hat{\varepsilon}_\lambda)\,dG_\varepsilon\leq\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_1-\hat{\varepsilon}_\lambda)\,dG_\varepsilon$ and $\int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon\leq\int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_2)\,dG_\varepsilon$. Substituting for the definitions of $h(\sigma_{1t})=\sigma_{1t}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_1-\varepsilon)\,dG_\varepsilon$ and $h(\sigma_{2t})=\sigma_{2t}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}(\hat{\varepsilon}_2-\varepsilon)\,dG_\varepsilon$, we get:

$$h(\sigma_{\lambda t})\leq\lambda h(\sigma_{1t})+(1-\lambda)h(\sigma_{2t})+\lambda\sigma_{1t}\left\{(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_1)\,G_\varepsilon(\hat{\varepsilon}_\lambda)\right\}+(1-\lambda)\sigma_{2t}\left\{(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_2)\,G_\varepsilon(\hat{\varepsilon}_\lambda)\right\}=$$
$$\lambda h(\sigma_{1t})+(1-\lambda)h(\sigma_{2t})+G_\varepsilon(\hat{\varepsilon}_\lambda)\left(\lambda\sigma_{1t}\,(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_1)+(1-\lambda)\sigma_{2t}\,(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_2)\right)=\lambda h(\sigma_{1t})+(1-\lambda)h(\sigma_{2t}),$$

where for the last equality we use equation (A.21) to show that

$$\lambda\sigma_{1t}\,(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_1)+(1-\lambda)\sigma_{2t}\,(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_2)=\hat{\varepsilon}_\lambda\,(\lambda\sigma_{1t}+(1-\lambda)\sigma_{2t})-\lambda\sigma_{1t}\hat{\varepsilon}_1-(1-\lambda)\sigma_{2t}\hat{\varepsilon}_2=0.$$

(b) $R^s_{t+1}>R^d_t\,(1-\gamma_t)$: it implies that $\hat{\varepsilon}_1<\hat{\varepsilon}_\lambda<\hat{\varepsilon}_2$

$$h(\sigma_{\lambda t})=(\lambda\sigma_{1t}+(1-\lambda)\sigma_{2t})\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_{\lambda t})}(\hat{\varepsilon}(\sigma_{\lambda t})-\varepsilon)\,dG_\varepsilon\right\}=$$
$$\lambda\sigma_{1t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon+\int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon\right\}+$$
$$(1-\lambda)\sigma_{2t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon-\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon\right\}=$$
$$\lambda\sigma_{1t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_2-\varepsilon)\,dG_\varepsilon+(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_1)\,G_\varepsilon(\hat{\varepsilon}_1)+\int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda}(\hat{\varepsilon}_\lambda-\varepsilon)\,dG_\varepsilon\right\}+$$
$$(1-\lambda)\sigma_{2t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}(\hat{\varepsilon}_2-\varepsilon)\,dG_\varepsilon+(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_2)\,G_\varepsilon(\hat{\varepsilon}_2)+\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2}(\varepsilon-\hat{\varepsilon}_\lambda)\,dG_\varepsilon\right\}\leq$$
$$\lambda\sigma_{1t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}(\hat{\varepsilon}_1-\varepsilon)\,dG_\varepsilon+(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_1)\,G_\varepsilon(\hat{\varepsilon}_1)+\int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda}(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_1)\,dG_\varepsilon\right\}+$$
$$(1-\lambda)\sigma_{2t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}(\hat{\varepsilon}_2-\varepsilon)\,dG_\varepsilon+(\hat{\varepsilon}_\lambda-\hat{\varepsilon}_2)\,G_\varepsilon(\hat{\varepsilon}_2)+\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2}(\hat{\varepsilon}_2-\hat{\varepsilon}_\lambda)\,dG_\varepsilon\right\},$$

where the inequality sign comes from $\int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \varepsilon)\, dG_\varepsilon \leq \int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1)\, dG_\varepsilon$ and $\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\varepsilon - \hat{\varepsilon}_\lambda)\, dG_\varepsilon \leq \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_2 - \hat{\varepsilon}_\lambda)\, dG_\varepsilon$. Substituting for the definitions of $h(\sigma_{1t}) = \sigma_{1t} \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1} (\hat{\varepsilon}_1 - \varepsilon)\, dG_\varepsilon$ and $h(\sigma_{2t}) = \sigma_{2t} \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_2 - \varepsilon)\, dG_\varepsilon$, we get:

$$h(\sigma_{\lambda t}) \leq \lambda h(\sigma_{1t}) + (1-\lambda) h(\sigma_{2t}) + \lambda \sigma_{1t} \left\{ (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1)\, G_\varepsilon(\hat{\varepsilon}_\lambda) \right\} + \\ (1-\lambda)\sigma_{2t} \left\{ (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2)\, G_\varepsilon(\hat{\varepsilon}_\lambda) \right\} = \lambda h(\sigma_{1t}) + (1-\lambda) h(\sigma_{2t}) + \\ G_\varepsilon(\hat{\varepsilon}_\lambda) \left(\lambda \sigma_{1t} (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1) + (1-\lambda)\sigma_{2t} (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2) \right) = \lambda h(\sigma_{1t}) + (1-\lambda) h(\sigma_{2t}),$$

where the last equality follows from the same reasoning employed in the previous case.

(c) $R^s_{t+1} = R^d_t (1 - \gamma_t)$. Hence, $\hat{\varepsilon}(\sigma_t) = 0$ and

$$h(\sigma_t) = \sigma_t \left[\int_{\underline{\varepsilon}}^{0} (0 - \varepsilon)\, dG_\varepsilon \right],$$

which is linear in σ_t

2. We found in step 1 that $h(\sigma_t,\, R^s_{t+1})$ is convex in σ_t for each $R^s_{t+1} \in \mathbb{R}$. Consider $P(\omega) \geq 0$ for each $R^s_{t+1}(\omega) \in \mathbb{R}$. Then the following function[3]:

$$\int_\Omega h\left(\sigma_t,\, R^s_{t+1}(\omega)\right) P(d\omega) = \mathbb{E}_t\, h(\sigma_t,\, R^s_{t+1}) \equiv H(\sigma_t)$$

is convex in σ_t. It follows directly from the linearity of the expectation operator which puts a non-negative weight on every realization of R^s_{t+1} and the fact that the sum of convex functions is a convex function. Therefore, $\Pi(\sigma_t)$ is convex in σ_t.

Since the expected dividend function is defined as the difference between expected net worth and the initial equity investment (in other words, it an affine transformation of expected net worth), that does not depend on σ_t, i.e., $\mathit{\Omega}(\sigma_t;\, l_t,\, d_t,\, e_t) \equiv \Pi(\sigma_t) - e_t$, the convexity of $\Pi(\sigma_t)$ guarantees that $\mathit{\Omega}(\sigma_t;\, l_t,\, d_t,\, e_t)$ is convex. □

A.2.7 Safe and Risky Banks

The results established in the preceding section imply that at any one time, depending on the state of the economy and the realization of aggregate shocks, only one type of bank may exist, either the risky bank or the safe bank. However, we have not been able to rule out analytically that risky and safe banks may coexist. Accordingly, we allow for this possibility in the numerical solution of the model. Nevertheless, in our model simulations, we have not found any such case.

We let μ_t denote the fraction of banks with risky portfolios (banks that choose $\sigma_t = \overline{\sigma}$) at date t; the remaining fraction $1 - \mu_t$ are safe banks ($\sigma_t = \underline{\sigma}$).

The fraction μ_t is endogenously determined by equity positions of households: we have $\mu_t = \frac{E^r_t}{E^r_t + E^s_t}$. At any point in time, the economy may be in a safe equilibrium (with $\mu_t = 0$), a risky equilibrium (with $\mu_t = 1$), or a mixed equilibrium (with $0 < \mu_t < 1$).

[3]Linearity in σ_t for one particular value of R^s_{t+1} can be considered as a weakly convex function, so it does not change the nature of the argument

Each bank within a group (safe or risky) is alike and solves the same maximization problem in which it chooses l_t^i, d_t^i, e_t^i according to its type $i \in \{s, r\}$.

A.3 The Non-Financial Firm's Problem

We consider the problem of safe firms before that of risky firms.

A.3.1 Safe Firms

Let π_{t+1}^s denote the revenue of a safe firm in period $t+1$ net of expenses:

$$\pi_{t+1}^s = y_{t+1}^s + (1-\delta)Q_t k_{t+1}^s - W_{t+1}h_{t+1}^s - R_{t+1}^s l_t^{f,s}.$$

In this notation, the problem of the safe firm is to

$$\max_{l_t^{f,s}, k_{t+1}^s} \mathbb{E}_t \left\{ \max_{h_{t+1}^s} \pi_{t+1}^s \right\}.$$

The first-order condition for $\max_{h_{t+1}^s} \pi_{t+1}^s$ is $\frac{\partial \pi_{t+1}^s}{\partial h_{t+1}^s} = 0$. It implies that

$$W_{t+1} = \frac{\partial y_{t+1}^s}{\partial h_{t+1}^s} = (1-\alpha)\frac{y_{t+1}^s}{h_{t+1}^s} = (1-\alpha) A_{t+1} \left(\frac{k_{t+1}^s}{h_{t+1}^s}\right)^{\alpha}, \tag{A.22}$$

$$h_{t+1}^s = (1-\alpha)\frac{y_{t+1}^s}{W_{t+1}} = (1-\alpha)\frac{A_{t+1}\left(k_{t+1}^s\right)^{\alpha}\left(h_{t+1}^s\right)^{1-\alpha}}{W_{t+1}}. \tag{A.23}$$

Accordingly, the safe firm's Lagrangian is:

$$\begin{aligned}\mathcal{L}^{\text{safe}} =& \mathbb{E}_t \left\{ A_{t+1}\left(k_{t+1}^s\right)^{\alpha}\left(h_{t+1}^s\right)^{1-\alpha} + (1-\delta)Q_{t+1}k_{t+1}^s - W_{t+1}h_{t+1}^s - R_{t+1}^s l_t^{f,s} \right\} + \\ & \lambda_{ht}^s \mathbb{E}_t \left\{ (1-\alpha)\frac{A_{t+1}\left(k_{t+1}^s\right)^{\alpha}\left(h_{t+1}^s\right)^{1-\alpha}}{W_{t+1}} - h_{t+1}^s \right\} + \lambda_{lt}^s \left(l_t^{f,s} - Q_t k_{t+1}^s \right).\end{aligned}$$

Notice that there is no expectation operator on the Lagrangian multipliers because those constraints hold under every state of nature. The problem implies the following first-order conditions

$$\begin{aligned}\frac{\partial \mathcal{L}^{\text{safe}}}{\partial l_t^{f,s}} &= -\mathbb{E}_t\left[R_{t+1}^s\right] + \lambda_{lt}^s = 0, \\ \frac{\partial \mathcal{L}^{\text{safe}}}{\partial k_{t+1}^s} &= \mathbb{E}_t\left[\alpha\frac{y_{t+1}^s}{k_{t+1}^s} + (1-\delta)Q_{t+1}\right] + \lambda_{ht}^s (1-\alpha)\alpha\mathbb{E}_t\left[\frac{A_{t+1}}{W_{t+1}}\left(\frac{k_{t+1}^s}{h_{t+1}^s}\right)^{\alpha-1}\right] - \lambda_{lt}^s Q_t = 0, \\ \frac{\partial \mathcal{L}^{\text{safe}}}{\partial h_{t+1}^s} &= (1-\alpha)\frac{A_{t+1}\left(k_{t+1}^s\right)^{\alpha}\left(h_{t+1}^s\right)^{1-\alpha}}{W_{t+1}} - W_{t+1} + \lambda_{ht}^s\left[(1-\alpha)^2\frac{A_{t+1}}{W_{t+1}}\left(\frac{k_{t+1}^s}{h_{t+1}^s}\right)^{\alpha} - 1\right] = 0.\end{aligned}$$

Combining $\frac{\partial \mathcal{L}^{\text{safe}}}{\partial h_{t+1}^s} = 0$ with equation (A.23) yields $\lambda_{ht}^s = 0$. Then, plugging $\frac{\partial \mathcal{L}^{\text{safe}}}{\partial l_t^{f,s}} = 0$ into $\frac{\partial \mathcal{L}^{\text{safe}}}{\partial k_{t+1}^s}$ for λ_{lt}^s, we get

$$\mathbb{E}_t\left[R_{t+1}^s\right]Q_t = \mathbb{E}_t\left[\alpha\frac{y_{t+1}^s}{k_{t+1}^s} + (1-\delta)Q_{t+1}\right].$$

Consider the zero-profit condition of the safe firm under all states of nature. Since the production function has constant returns to scale,

$$y_{t+1}^s = \frac{\partial y_{t+1}^s}{\partial k_{t+1}^s} k_{t+1}^s + \frac{\partial y_{t+1}^s}{\partial h_{t+1}^s} h_{t+1}^s = \alpha A_{t+1} \left(\frac{k_{t+1}^s}{h_{t+1}^s} \right)^{\alpha-1} k_{t+1}^s + W_{t+1} h_{t+1}^s,$$

where we use equation (A.23) to substitute for W_{t+1} in the last equality. Plugging the expression of y_{t+1}^s into $\pi_{t+1}^s = 0$ and using $Q_t k_{t+1}^s = l_t^{f,s}$, we find that:

$$\alpha A_{t+1} \left(\frac{k_{t+1}^s}{h_{t+1}^s} \right)^{\alpha-1} k_{t+1}^s + (1-\delta) Q_{t+1} k_{t+1}^s - R_{t+1}^s Q_t k_{t+1}^s = 0.$$

Since $k_{t+1}^s > 0$, we can divide by k_{t+1}^s to get

$$R_{t+1}^s Q_t = \alpha A_{t+1} \left(\frac{k_{t+1}^s}{h_{t+1}^s} \right)^{\alpha-1} + (1-\delta) Q_{t+1} \tag{A.24}$$

under all states of nature. This condition implies

$$R_t^s Q_{t-1} = \alpha A_t \left(\frac{k_t^s}{h_t^s} \right)^{\alpha-1} + (1-\delta) Q_t.$$

A.3.2 Risky Firms

Let π_{t+1}^r denote the revenue of a risky firm in period $t+1$ net of expenses:

$$\pi_{t+1}^r = y_{t+1}^r + (1-\delta) Q_t k_{t+1}^r - W_{t+1} h_{t+1}^r - R_{t+1}^r l_t^{f,r}.$$

In this notation, the problem of the risky firm is to

$$\max_{l_t^{f,r}, k_{t+1}^r} \mathbb{E}_t \left\{ \max_{h_{t+1}^r} \pi_{t+1}^r \right\}.$$

The first-order condition for $\max_{h_{t+1}^r} \pi_{t+1}^r$ is $\frac{\partial \pi_{t+1}^r}{\partial h_{t+1}^r} = 0$. It implies that

$$W_{t+1} = \frac{\partial y_{t+1}^r}{\partial h_{t+1}^r} = (1-\alpha) A_{t+1} \left(\frac{k_{t+1}^r}{h_{t+1}^r} \right)^{\alpha}, \tag{A.25}$$

$$h_{t+1}^r = (1-\alpha) \frac{A_{t+1} \left(k_{t+1}^r\right)^{\alpha} \left(h_{t+1}^r\right)^{1-\alpha}}{W_{t+1}}. \tag{A.26}$$

Accordingly, the risky firm's Lagrangian is:

$$\begin{aligned}\mathcal{L}^{\text{risky}} =& \mathbb{E}_t \left[A_{t+1} \left(k_{t+1}^r\right)^{\alpha} \left(h_{t+1}^r\right)^{1-\alpha} + \varepsilon_{t+1} k_{t+1}^r + (1-\delta) Q_{t+1} k_{t+1}^r - W_{t+1} h_{t+1}^r - R_{t+1}^r l_t^{f,r} \right] + \\ & \lambda_{ht}^r \mathbb{E}_t \left[(1-\alpha) \frac{A_{t+1} \left(k_{t+1}^r\right)^{\alpha} \left(h_{t+1}^r\right)^{1-\alpha}}{W_{t+1}} - h_{t+1}^r \right] + \lambda_{lt}^r \left(l_t^{f,r} - Q_t k_{t+1}^r \right).\end{aligned}$$

Notice that there is no expectation operator on the Lagrangian multipliers because those constraints hold under every state of nature. The problem implies the following first-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}^{\text{risky}}}{\partial l_t^{f,r}} &= -\mathbb{E}_t\left[R_{t+1}^r\right] + \lambda_{lt}^r = 0,\\
\frac{\partial \mathcal{L}^{\text{risky}}}{\partial k_{t+1}^r} &= \mathbb{E}_t\left[\alpha A_{t+1}\left(\frac{k_{t+1}^r}{h_{t+1}^r}\right)^{\alpha-1} + \varepsilon_{t+1} + (1-\delta)Q_{t+1}\right] + \\
&\quad \lambda_{ht}^r \mathbb{E}_t\left[\alpha\left(1-\alpha\right)\frac{A_{t+1}}{W_{t+1}}\left(\frac{k_{t+1}^r}{h_{t+1}^r}\right)^{\alpha-1}\right] - \lambda_{lt}^r Q_t = 0,\\
\frac{\partial \mathcal{L}^{\text{risky}}}{\partial h_{t+1}^r} &= \left(1-\alpha\right)A_{t+1}\left(\frac{k_{t+1}^r}{h_{t+1}^r}\right)^{\alpha} - W_{t+1} + \lambda_{ht}^r\left[\left(1-\alpha\right)^2\frac{A_{t+1}}{W_{t+1}}\left(\frac{k_{t+1}^r}{h_{t+1}^r}\right)^{\alpha} - 1\right] = 0.
\end{aligned}$$

Equation (A.25) together with $\frac{\partial \mathcal{L}^{\text{risky}}}{\partial h_{t+1}^r} = 0$ yield $\lambda_{ht}^r = 0$. Plugging $\frac{\partial \mathcal{L}^{\text{risky}}}{\partial l_t^{f,r}} = 0$ into $\frac{\partial \mathcal{L}^{\text{risky}}}{\partial k_{t+1}^r}$ for λ_{lt}^r, we get

$$\mathbb{E}_t\left[R_{t+1}^r\right]Q_t = \mathbb{E}_t\left[\alpha A_{t+1}\left(\frac{k_{t+1}^r}{h_{t+1}^r}\right)^{\alpha-1} + (1-\delta)Q_{t+1} + \varepsilon_{t+1}\right].$$

Combining equation (A.22) with equation (A.25):

$$\frac{k_{t+1}^s}{h_{t+1}^s} = \frac{k_{t+1}^r}{h_{t+1}^r} \tag{A.27}$$

under all states of nature. But remember that the first-order condition of the safe firm implies

$$\mathbb{E}_t\left[R_{t+1}^s\right]Q_t = \mathbb{E}_t\left[\alpha A_{t+1}\left(\frac{k_{t+1}^s}{h_{t+1}^s}\right)^{\alpha-1} + (1-\delta)Q_{t+1}\right].$$

Therefore

$$\mathbb{E}_t\left[R_{t+1}^s\right]Q_t = \mathbb{E}_t\left[R_{t+1}^s Q_t + \varepsilon_{t+1}\right].$$

Consider the zero-profit condition of the risky firm under all states of nature.

$$\begin{aligned}
\pi_{t+1}^r = y_{t+1}^r + (1-\delta)Q_t k_{t+1}^r - W_{t+1}h_{t+1}^r - R_{t+1}^r l_t^{f,r} = \\
y_{t+1}^r + (1-\delta)Q_t k_{t+1}^r - \left(1-\alpha\right)A_{t+1}\left(k_{t+1}^r\right)^{\alpha}\left(h_{t+1}^r\right)^{1-\alpha} - R_{t+1}^r l_t^{f,r} = \\
\alpha A_{t+1}\left(k_{t+1}^r\right)^{\alpha}\left(h_{t+1}^r\right)^{1-\alpha} + \varepsilon_{t+1}k_{t+1}^r + (1-\delta)Q_t k_{t+1}^r - R_{t+1}^r l_t^{f,r} = \\
\alpha A_{t+1}\left(\frac{k_{t+1}^r}{h_{t+1}^r}\right)^{\alpha-1} k_{t+1}^r + \varepsilon_{t+1}k_{t+1}^r + (1-\delta)Q_t k_{t+1}^r - R_{t+1}^r l_t^{f,r} = 0,
\end{aligned}$$

where we use equation (A.26) to substitute for $W_{t+1}h_{t+1}^r$. Using equation (A.24) together with equation (A.27), we can express

$$\alpha A_{t+1}\left(\frac{k_{t+1}^r}{h_{t+1}^r}\right)^{\alpha-1} = R_{t+1}^s Q_t - (1-\delta)Q_{t+1},$$

that holds under all states of nature. Plugging it into the zero-profit condition and using $Q_t k_{t+1}^r = l_t^{f,r}$, we find that:

$$R_{t+1}^s Q_t k_{t+1}^r - (1-\delta)Q_{t+1}k_{t+1}^r + \varepsilon_{t+1}k_{t+1}^r + (1-\delta)Q_t k_{t+1}^r - R_{t+1}^r Q_t k_{t+1}^r = 0.$$

Since $k^r_{t+1} > 0$, we can divide by k^r_{t+1} to get

$$R^r_{t+1}Q_t = R^s_{t+1}Q_t + \varepsilon_{t+1}$$

under all states of nature. This condition implies

$$R^r_t Q_{t-1} = R^s_t Q_{t-1} + \varepsilon_t.$$

A.3.3 Aggregating Across Firms

Here we show that we can aggregate individual firms into two representative firms. Let $k^i_{j,t}$ denote the capital chosen by firm i that is financed by borrowing from bank j. Both i and j lie within the continuum of measure 1 of banks and firms, respectively. In this notation, equation (A.27) is written as

$$\frac{k^i_{j,t+1}}{h^i_{j,t+1}} = \frac{k_{t+1}}{h_{t+1}}, \tag{A.28}$$

for all $j \in [0,1]$ and $i \in [0,1]$. Each firm chooses the same capital-to-labor ratio independently of the type of bank it borrows from.

Note that σ_t is the fraction of risky firms at date t; the remaining fraction $1-\sigma_t$ of firms are safe firms. Let's index firms as follows: firm j_1, with $j_1 \in [0,\sigma_t]$, can only access a risky technology subject to both aggregate and idiosyncratic shocks; firm j_2, with $j_2 \in [\sigma_t, 1]$ has access to a safe production technology subject to aggregate shocks only. Since there are no equilibria with $\underline{\sigma} < \sigma_t < \overline{\sigma}$, the fraction of risky firms is linked to the fraction of banks with risky portfolios as follows:

$$\sigma_t = (1-\mu_t)\,\underline{\sigma} + \mu_t\overline{\sigma}.$$

Define the following objects: Let $K^s_{s,t+1} = \int_{\sigma_t}^1 \int_{\mu_t}^1 k^i_{j,t+1} dj di$ be the total capital allocated to the safe technology and financed by borrowing from the banks that choose a fraction $\underline{\sigma}$ of risky projects. Let $K^s_{r,t+1} = \int_{\sigma_t}^1 \int_0^{\mu_t} k^i_{j,t+1} dj di$ be the total capital allocated to the safe technology and financed by borrowing from the banks that choose a fraction $\overline{\sigma}$ of risky projects. We let K^s_{t+1} denote the total capital allocated to the safe technology. Thus,

$$K^s_{t+1} = \int_{\sigma_t}^{1}\int_{0}^{1} k^i_{j,t+1} dj di = K^s_{s,t+1} + K^s_{r,t+1},$$

Let $K^r_{s,t+1} = \int_0^{\sigma_t} \int_{\mu_t}^1 k^i_{j,t+1} dj di$ be the total capital allocated to the risky technology and financed by borrowing from the banks that choose a fraction $\underline{\sigma}$ of risky projects. Let $K^r_{r,t+1} = \int_0^{\sigma_t} \int_0^{\mu_t} k^i_{j,t+1} dj di$ be the total capital allocated to the safe technology and financed by borrowing from the banks that choose a fraction $\overline{\sigma}$ of risky projects. We let K^r_{t+1} denote the total capital allocated to the risky technology. Thus,

$$K^r_{t+1} = \int_{0}^{\sigma_t}\int_{0}^{1} k^i_{j,t+1} dj di = K^r_{s,t+1} + K^r_{r,t+1},$$

The same upper and lower case notation applies to labor, i.e. $H^s_{s,t+1} = \int_{\sigma_t}^1 \int_{\mu_t}^1 h^i_{j,t+1} dj di$; $H^s_{r,t+1} = \int_{\sigma_t}^1 \int_0^{\mu_t} h^i_{j,t+1} dj di$; $H^r_{s,t+1} = \int_0^{\sigma_t} \int_{\mu_t}^1 h^i_{j,t+1} dj di$; $H^r_{r,t+1} = \int_0^{\sigma_t} \int_0^{\mu_t} h^i_{j,t+1} dj di$.

Safe representative firm produces:

$$Y_t^s = \int_{\sigma_{t-1}}^{1} \int_{0}^{1} A_t \left(k_{j,t}^i\right)^{\alpha} \left(h_{j,t}^i\right)^{1-\alpha} djdi = \int_{\sigma_{t-1}}^{1} \int_{0}^{1} F\left(k_{j,t}^i, h_{j,t}^i\right) djdi =$$

Using that the technology has Constant Returns to Scale:

$$= \int_{\sigma_{t-1}}^{1} \int_{0}^{1} \left[F_{k_{j,t}^i}\left(k_{j,t}^i, h_{j,t}^i\right) k_{j,t}^i + F_{h_{j,t}^i}\left(k_{j,t}^i, h_{j,t}^i\right) h_{j,t}^i\right] djdi =$$

where $F_{k_{j,t}^i}\left(k_{j,t}^i, h_{j,t}^i\right)$ and $F_{h_{j,t}^i}\left(k_{j,t}^i, h_{j,t}^i\right)$ denote the partial derivative of $F\left(k_{j,t}^i, h_{j,t}^i\right)$ with respect to $k_{j,t}^i$ and $h_{j,t}^i$, respectively. Since these partial derivatives are homogeneous of degree zero, we can express them in term of capital-labor ratio, i.e.

$$= \int_{\sigma_{t-1}}^{1} \int_{0}^{1} \left[f_{k_{j,t}^i}\left(\frac{k_{j,t}^i}{h_{j,t}^i}\right) k_{j,t}^i + f_{h_{j,t}^i}\left(\frac{k_{j,t}^i}{h_{j,t}^i}\right) h_{j,t}^i\right] djdi = \text{Plugging equation (A.28)} =$$

$$= \int_{\sigma_{t-1}}^{1} \int_{0}^{1} \left[f_{k_t}\left(\frac{k_t}{h_t}\right) k_{j,t}^i + f_{h_t}\left(\frac{k_t}{h_t}\right) h_{j,t}^i\right] djdi =$$

$$f_{k_t}\left(\frac{k_t}{h_t}\right)\left[\int_{\sigma_t}^{1} \int_{0}^{1} k_{j,t}^i djdi\right] + f_{h_t}\left(\frac{k_t}{h_t}\right)\left[\int_{\sigma_t}^{1} \int_{0}^{1} h_{j,t}^i djdi\right] = f_{k_t}\left(\frac{k_t}{h_t}\right) K_t^s + f_{h_t}\left(\frac{k_t}{h_t}\right) H_t^s =$$

Since $\frac{K_{s,t}^s}{H_{s,t}^s} = \frac{K_{r,t}^s}{H_{r,t}^s} = \frac{k_t}{h_t}$,then $\frac{K_t^s}{H_t^s}\frac{h_t}{k_t} = \left(\frac{K_{s,t}^s + K_{r,t}^s}{H_{s,t}^s + H_{r,t}^s}\right)\frac{H_{r,t}^s}{K_{r,t}^s} = 1$. Therefore $\frac{K_t^s}{H_t^s} = \frac{k_t}{h_t}$.

$$= f_{K_t^s}\left(\frac{K_t^s}{H_t^s}\right) K_t^s + f_{H_t^s}\left(\frac{K_t^s}{H_t^s}\right) H_t^s.$$

Hence, we obtain that

$$Y_t^s = A_t \left(K_t^s\right)^{\alpha} \left(H_t^s\right)^{1-\alpha}. \tag{A.29}$$

Moving to the risky representative firm,

$$Y_t^r = \int_{0}^{\sigma_{t-1}} \int_{0}^{1} \left[A_t \left(k_{j,t}^i\right)^{\alpha} \left(h_{j,t}^i\right)^{1-\alpha} + \varepsilon_{j,t}^i k_{j,t}^i\right] djdi = \int_{0}^{\sigma_{t-1}} \int_{0}^{1} F\left(k_{j,t}^i, h_{j,t}^i\right) djdi + \int_{0}^{\sigma_{t-1}} \int_{0}^{1} \varepsilon_{j,t}^i k_{j,t}^i djdi.$$

Note that similar steps described above apply to the first term in the summation, so that $\int_0^{\sigma_{t-1}} \int_0^1 F\left(k_{j,t}^i, h_{j,t}^i\right) djdi = A_t \left(K_t^r\right)^{\alpha} \left(H_t^r\right)^{1-\alpha}$. To express the second term, notice that $\int_0^{\sigma_{t-1}} \int_0^1 \varepsilon_{j,t}^i k_{j,t}^i djdi = -\xi$. Moreover, since each risky firm solves the same maximization problem, it chooses the same amount of capital independently of the type of bank it borrows from. Therefore, $\int_0^{\sigma_{t-1}} \int_0^1 \varepsilon_{j,t}^i k_{j,t}^i djdi = -\xi K_t^r$. Hence,

$$Y_t^r = A_t \left(K_t^r\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} - \xi K_t^r. \tag{A.30}$$

A.3.4 Capital Producing Firms

At the end of period t, goods producing firms sell their capital to competitive capital producing firms. Letting I_t^g denote gross investment, the evolution of capital follows

$$I_t = \eta_t \left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g,$$

where η_t is a shock to investment-specific technology (ISP), and ϕ is a measure of the severity of investment adjustment costs. The aggregate capital stock evolves according to

$$K_{t+1}^s + K_{t+1}^r = I_t + (1-\delta)\left(K_t^s + K_t^r\right).$$

The capital producing firms are owned by households, and solve the problem

$$\max_{I_{t+i}^g} \mathbb{E}_t \sum_{i=0}^{\infty} \psi_{t,t+i} \left\{ \eta_{t+i} Q_{t+i} \left[1 - \frac{\phi}{2}\left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1\right)^2\right] I_{t+i}^g - I_{t+i}^g \right\},$$

where $\psi_{t,t+i} = \beta \frac{\lambda_{ct+i}}{\lambda_{ct}}$ is the stochastic discount factor of the households.

$$\eta_t Q_t \left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - \eta_t Q_t \phi \left(\frac{I_t^g}{I_{t-1}^g} - 1\right) \frac{I_t^g}{I_{t-1}^g} - 1 + \beta \mathbb{E}_t \left[\eta_{t+1} \frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} \phi \left(\frac{I_{t+1}^g}{I_t^g} - 1\right) \frac{I_{t+1}^g}{\left(I_t^g\right)^2} I_{t+1}^g\right] = 0.$$

A.4 The Government

The government levies the tax to fully compensate for the loss to the deposit insurance fund due to rescue of defaulted banks.

$$T_t = -\int_{-\infty}^{\left(\frac{R_{t-1}^d D_{t-1}}{\sigma_{t-1} L_{t-1}} - \frac{R_t^s}{\sigma_{t-1}}\right) Q_{t-1}} \left(\left(R_t^s + \frac{\sigma_{t-1}\varepsilon_t}{Q_{t-1}}\right) L_{t-1} - R_{t-1}^d D_{t-1}\right) \mathrm{d}G(\varepsilon_t) =$$

$$-\left[\int_{-\infty}^{\infty} \left(\left(R_t^s + \frac{\sigma_{t-1}\varepsilon_t}{Q_{t-1}}\right) L_{t-1} - R_{t-1}^d D_{t-1}\right) \mathrm{d}G(\varepsilon_t) - \int_{\left(\frac{R_{t-1}^d D_{t-1}}{\sigma_{t-1} L_{t-1}} - \frac{R_t^s}{\sigma_{t-1}}\right) Q_{t-1}}^{\infty} \left(\left(R_t^s + \frac{\sigma_{t-1}\varepsilon_t}{Q_{t-1}}\right) L_{t-1} - R_{t-1}^d D_{t-1}\right) \mathrm{d}G(\varepsilon_t)\right] =$$

Note that in the square bracket the first term equals $\left(R_t^s - \frac{\sigma_{t-1}\xi}{Q_{t-1}}\right) L_{t-1} + R_{t-1}^d D_{t-1}$. We have already calculated the second term. Therefore,

$$\begin{aligned} T_t \quad = \quad & \frac{\sigma_{t-1}L_{t-1}}{Q_{t-1}} \frac{\tau_t}{\sqrt{2\pi}} \exp\left(-\left(\frac{R_{t-1}^d\left(1-\gamma_{t-1}\right)Q_{t-1} - R_t^s Q_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_t}\right)^2\right) - \left(R_t^s - \frac{\sigma_{t-1}\xi}{Q_{t-1}}\right) L_{t-1} + R_{t-1}^d D_{t-1} + \\ & \frac{1}{2} L_{t-1}\left(R_t^s - \frac{\sigma_{t-1}\xi}{Q_{t-1}} - \left(1-\gamma_{t-1}\right) R_{t-1}^d\right)\left[1 - \operatorname{erf}\left(\frac{R_{t-1}^d\left(1-\gamma_{t-1}\right)Q_{t-1} - R_t^s Q_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_t}\right)\right], \end{aligned}$$

which can be rewritten as

$$\begin{aligned} T_t \quad = \quad & \frac{\sigma_{t-1}L_{t-1}}{Q_{t-1}} \frac{\tau_t}{\sqrt{2\pi}} \exp\left(-\left(\frac{R_{t-1}^d\left(1-\gamma_{t-1}\right)Q_{t-1} - R_t^s Q_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_t}\right)^2\right) - \\ & \frac{1}{2}\left(R_t^s L_{t-1} - \frac{\sigma_{t-1}\xi}{Q_{t-1}} L_{t-1} - R_{t-1}^d D_{t-1}\right)\left[1 + \operatorname{erf}\left(\frac{R_{t-1}^d\left(1-\gamma_{t-1}\right)Q_{t-1} - R_t^s Q_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_t}\right)\right]. \end{aligned} \tag{A.31}$$

A.5 Resource Constraints

The aggregate loans to the (representative) safe firm come from two sources: 1) from all safe banks (of measure $1-\mu_t$) that allocate $1-\underline{\sigma}$ share of their loan portfolio to safe projects and 2) from all risky banks (of measure μ_t) that allocate $1-\overline{\sigma}$ share of their loan portfolio to safe projects. Therefore, the equilibrium conditions linking our bank-level and firm-level variables representing loans are

$$Q_t K_{t+1}^s = \left(1-\underline{\sigma}\right)\left(1-\mu_t\right) l_t^s + \left(1-\overline{\sigma}\right)\mu_t l_t^r.$$

Similarly,

$$Q_t K_{t+1}^r = \underline{\sigma}\left(1-\mu_t\right) l_t^s + \overline{\sigma}\mu_t l_t^r.$$

The aggregate bank loans are linked to the individual bank loans by: $L_t^r = \mu_t l_t^r$ and $L_t^s = (1-\mu_t) l_t^s$. Therefore, we can describe the latter two equations by using aggregate loans

$$\begin{aligned} Q_t K_{t+1}^s &= \left(1-\underline{\sigma}\right) L_t^s + \left(1-\overline{\sigma}\right) L_t^r, \\ Q_t K_{t+1}^r &= \underline{\sigma} L_t^s + \overline{\sigma} L_t^r. \end{aligned}$$

The equity positions taken by households, in turn, determine the equity positions of individual banks: $E_t^r = \mu_t e_t^r$ and $E_t^s = (1-\mu_t) e_t^s$. The returns on the equity positions taken by households at date t are linked to the dividends paid by banks at date $t+1$. We have:

$$\begin{aligned} \mathbb{E}_t^r R_{t+1}^{e,r} &= \left(\omega_1^r + \omega_2^r\right) L_t^r, \\ \mathbb{E}_t^s R_{t+1}^{e,s} &= \left(\omega_1^s + \omega_2^s\right) L_t^s, \end{aligned}$$

where we use the fact that $\max\left[nw_{t+1}^r, 0\right]$ is linear in loans; ω_1 and ω_2 were defined in equations (26) and (27). Deposits held by households are issued by (safe and risky) banks: $D_t = D_t^s + D_t^r$ where $D_t^s = L_t^s - E_t^s$ and $D_t^r = L_t^r - E_t^r$.

The equilibrium conditions linking our aggregate and individual firm-specific variables are straightforward, but cumbersome in terms of notation. We state the conditions in Appendix B. The market-clearing conditions for labor, capital, and goods are

$$H_t^s + H_t^r = 1,$$

$$K_t^s + K_t^r = K_t,$$

and

$$Y_t^s + Y_t^r = C_t + I_t^g.$$

B List of Equilibrium Conditions

From the first-order conditions for the household problem, derived in Section A.1

$$(C_t - \kappa C_{t-1})^{-\varsigma_c} - \beta\kappa \mathbb{E}_t \left(C_{t+1} - \kappa C_t\right)^{-\varsigma_c} - \lambda_{ct} = 0 \tag{B.1}$$

$$\varsigma_0 D_t^{-\varsigma_d} - \lambda_{ct} + \mathbb{E}_t \beta \lambda_{ct+1} R_t^d = 0, \tag{B.2}$$

$$-\lambda_{ct} + \mathbb{E}_t \beta \lambda_{ct+1} R_{t+1}^{e,s} + \zeta_t^s = 0, \tag{B.3}$$

$$-\lambda_{ct} + \mathbb{E}_t \beta \lambda_{ct+1} R_{t+1}^{e,r} + \zeta_t^r = 0. \tag{B.4}$$

From Section A.2.6, we know that we need to track two types of banks, safe and risky banks:

$$\sigma_t^s = \underline{\sigma}, \tag{B.5}$$

$$\sigma_t^r = \overline{\sigma}. \tag{B.6}$$

From the bank problem in Section A.2, for $\forall i \in \{s, r\}$

$$l_t^i = d_t^i + e_t^i. \tag{B.7}$$

From the proof that capital requirements always bind in Section A.2.2, for $\forall i \in \{s, r\}$

$$e_t^i = \gamma_t l_t^i. \tag{B.8}$$

From the combined first-order conditions for banks in Section A.2.3, for $\forall i \in \{s, r\}$

$$\gamma_t - \chi_{2t}^i = \mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\frac{\sigma_t^i}{Q_t} \frac{\tau_t}{\sqrt{2\pi}} \exp\left(- \left(\frac{\left(R_t^d(1-\gamma_t) - R_{t+1}^s\right)Q_t + \xi\sigma_t^i}{\sigma_t^i \sqrt{2}\tau_t} \right)^2 \right) + \right. \right. \tag{B.9}$$

$$\left. \left. \frac{1}{2}\left(R_{t+1}^s - \frac{\sigma_t^i \xi}{Q_t} - R_t^d(1-\gamma_t) \right) \left[1 - \operatorname{erf}\left(\frac{\left(R_t^d(1-\gamma_t) - R_{t+1}^s\right)Q_t + \xi\sigma_t^i}{\sigma_t^i \sqrt{2}\tau_t} \right) \right] \right] \right\},$$

$$\chi_{2t}^i l_t^i = 0. \tag{B.10}$$

From the zero-profit condition for banks derived in Section A.2.4, for $\forall i \in \{s, r\}$

$$R_{t+1}^{e,i} = \frac{1}{\gamma_t} \left\{ \frac{\sigma_t^i}{Q_t} \frac{\tau_t}{\sqrt{2\pi}} \exp\left(- \left(\frac{\left(R_t^d(1-\gamma_t) - R_{t+1}^s\right)Q_t + \xi\sigma_t^i}{\sigma_t^i \sqrt{2}\tau_t} \right)^2 \right) + \right. \tag{B.11}$$

$$\left. \frac{1}{2}\left(R_{t+1}^s - \frac{\sigma_t^i \xi}{Q_t} - R_t^d(1-\gamma_t) \right) \left[1 - \operatorname{erf}\left(\frac{\left(R_t^d(1-\gamma_t) - R_{t+1}^s\right)Q_t + \xi\sigma_t^i}{\sigma_t^i \sqrt{2}\tau_t} \right) \right] \right\}.$$

Definition of the share of risky banks, from the aggregation across banks in Section A.2.7

$$\mu_t = \frac{E_t^r}{E_t^s + E_t^r}. \tag{B.12}$$

From the problem of safe firms in Section A.3.1

$$R_t^s = \frac{\alpha A_t}{Q_t} \left(\frac{K_t^s}{H_t^s} \right)^{\alpha - 1} + (1 - \delta) \frac{Q_{t+1}}{Q_t}. \tag{B.13}$$

From the problem of risky firms in Section A.3.2

$$R_t^r = R_t^s + \frac{\varepsilon_t}{Q_{t-1}}. \tag{B.14}$$

From Equation A.22 in Section A.3.1 and Section A.3.3

$$W_t = (1-\alpha)\frac{Y_t^s}{H_t^s}. \tag{B.15}$$

From the aggregation across safe and risky firms in Section A.3.3

$$\frac{K_t^s}{H_t^s} = \frac{K_t^r}{H_t^r}, \tag{B.16}$$

$$Y_t^s = A_t\left(K_t^s\right)^{\alpha}\left(H_t^s\right)^{1-\alpha}, \tag{B.17}$$

$$Y_t^r = A_t\left(K_t^r\right)^{\alpha}\left(H_t^r\right)^{1-\alpha} - \xi K_t^r. \tag{B.18}$$

From the problem of capital producing firms in Section A.3.4

$$K_{t+1} = I_t + (1-\delta)K_t, \tag{B.19}$$

$$I_t = \eta_t\left[1-\frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g}-1\right)^2\right]I_t^g, \tag{B.20}$$

$$\eta_t Q_t\left[1-\frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g}-1\right)^2\right] - \eta_t Q_t\phi\left(\frac{I_t^g}{I_{t-1}^g}-1\right)\frac{I_t^g}{I_{t-1}^g} - 1 + \tag{B.21}$$

$$\beta\mathbb{E}_t\left[\eta_{t+1}\frac{\lambda_{ct+1}}{\lambda_{ct}}Q_{t+1}\phi\left(\frac{I_{t+1}^g}{I_t^g}-1\right)\frac{I_{t+1}^g}{\left(I_t^g\right)^2}I_{t+1}^g\right] = 0.$$

The tax for the deposit insurance scheme is derived in Section A.4

$$T_t = L_{t-1}\left\{\frac{\sigma_{t-1}}{Q_{t-1}}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_{t-1}^d(1-\gamma_{t-1})-R_t^s\right)Q_{t-1}+\xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_t}\right)^2\right) - \right. \tag{B.22}$$

$$\left.\frac{1}{2}\left(R_t^s - R_{t-1}^d\left(1-\gamma_{t-1}\right) - \frac{\xi\sigma_{t-1}}{Q_{t-1}}\right)\left[1+\operatorname{erf}\left(\frac{\left(R_{t-1}^d(1-\gamma_{t-1})-R_t^s\right)Q_{t-1}+\xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_t}\right)\right]\right\}.$$

Using the resource constraints from Section A.5

$$L_t^s = \left(1-\mu_t\right)l_t^s, \tag{B.23}$$

$$L_t^r = \mu_t l_t^r, \tag{B.24}$$

$$E_t^i = \gamma_t L_t^i \quad \forall i\in\{s,r\}, \tag{B.25}$$

$$L_t^i = D_t^i + E_t^i \quad \forall i\in\{s,r\}, \tag{B.26}$$

$$Q_t K_{t+1}^s = \left(1-\underline{\sigma}\right)L_t^s + \left(1-\overline{\sigma}\right)L_t^r, \tag{B.27}$$

$$Q_t K_{t+1}^r = \underline{\sigma}L_t^s + \overline{\sigma}L_t^r, \tag{B.28}$$

$$D_t = D_t^s + D_t^r, \tag{B.29}$$

$$K_t = K_t^s + K_t^r, \tag{B.30}$$

$$H_t^s + H_t^r = 1, \tag{B.31}$$

$$Y_t^s + Y_t^r = C_t + I_t^g. \tag{B.32}$$

From the first-order conditions of the bank problem in Section A.2.1, the Lagrange multipliers χ_{2t}^{i} on the loan constraints $l_t^s > 0$, where $i \in \{s, r\}$, govern the transition between different regimes, demarcated as follows:

1. safe regime: $\chi_{2t}^{s} = 0$, $l_t^s > 0$, $\chi_{2t}^{r} > 0$, and $l_t^r = 0$;
2. risky regime: $\chi_{2t}^{s} > 0$, $l_t^s = 0$, $\chi_{2t}^{r} = 0$, and $l_t^r > 0$;
3. mixed regime: $\chi_{2t}^{s} = 0$, $l_t^s > 0$, $\chi_{2t}^{r} = 0$, and $l_t^r > 0$.

These conditions, together with binding capital requirements, also imply whether the inequality constraints on the household bank equity holdings bind or not. Accordingly,

1. safe regime: $\zeta_t^s = 0$, $E_t^s > 0$, $\zeta_t^r > 0$, $E_t^r = 0 \implies \mu_t = 0$;
2. risky regime: $\zeta_t^s > 0$, $E_t^s = 0$, $\zeta_t^r = 0$, and $E_t^r > 0 \implies \mu_t = 1$;
3. mixed regime: $\zeta_t^s = 0$, $E_t^s > 0$, $\zeta_t^r = 0$, and $E_t^r > 0 \implies 0 < \mu_t < 1$;

and remember that μ_t is the share of risky bank equity in total bank equity. Notice that in the safe regime, $l_t^r = 0$ and the binding capital requirement make $E_t^r = 0$ redundant; in the risky regime, $l_t^s = 0$ and the binding capital requirement make $E_t^s = 0$ redundant; and in the mixed regime, the fact that $\chi_{2t}^{r} = 0$ and that $\zeta_t^r = 0$ together with Equation B.4 and Equation B.11, make Equation B.9 redundant.

Finally, in the numerical implementation of the model, we use a guess-and-verify approach to determine the regime sequence expected to occur after the realization of the aggregate shocks period by period (see (Guerrieri & Iacoviello, 2015)).In the iterative procedure to determine the regime guesses, the following conditions prompt a new guess:

1. safe regime exit: $l_t^s < 0$,
2. risky regime exit: $l_t^r < 0$,
3. mixed regime exit: $l_t^s < 0$ or $l_t^r < 0$.

C Discussion of the Excessive Risk-Taking Mechanism

Following our results derived in Appendix A.2.5, we can decompose expected dividends into the following components:

$$\Omega\left(\mu_t;\ l_t, d_t, e_t\right) = \mathbb{E}_t\left\{\psi_{t,t+1} l_t\left[\omega_1 + \omega_2 -\right]\right\} - \gamma_t l_t,$$

where

$$[\omega_1 + \omega_2] = \left[\underbrace{\left(R^s_{t+1} - R^d_t\left(1-\gamma_t\right) - \frac{\xi\sigma_t}{Q_t}\right)\underbrace{\left(1 - G(\varepsilon^*_{t+1})\right)}_{\text{non-defaulted}}}_{\substack{\omega_1 \equiv \text{returns from a loan} \\ \text{portfolio with riskiness } \sigma_t}} + \underbrace{\left(\frac{\sigma_t}{Q_t}\right)\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\varepsilon^*_{t+1}+\xi}{\tau_t\sqrt{2}}\right)^2\right)}_{\substack{\omega_2 \equiv \text{bonus from} \\ \text{projects volatility}}}\right],$$

and the cutoff point ε^*_{t+1} is defined by $R^d_t\left(1-\gamma_t\right)Q_t - R^s_{t+1}Q_t = \sigma_t\varepsilon^*_{t+1}$.

The first component, ω_1, represents loan returns of riskiness σ_t controlling for the variance of idiosyncratic shock (when τ is taken as given). The bank trades off the benefits from limited liability and deposit insurance with a smaller profitability of riskier projects. The term $\frac{\xi\sigma_t}{Q_t}$ reflects, in expectation, the reduction of loan returns for the bank holding σ_t share of risky projects. The bank receives net income on loans, $R^s_{t+1} - R^d_t\left(1-\gamma_t\right) - \frac{\xi\sigma_t}{Q_t}$, if it does not default on deposits which happens with probability $1 - G(\varepsilon^*_{t+1})$. If the bank defaults, it gets zero, i.e. $0 \cdot G(\varepsilon^*_{t+1})$ which is not shown in the expression explicitly.

The second component, ω_2, represents the extra effect of σ_t on expected net worth owing to more dispersed returns from projects. In fact, ω_2 is strictly increasing in τ: the bank views projects as a call option the value of which rises with volatility associated with higher upside. Limited liability bounds the payoff to zero in the worst case scenario.

Risk-taking incentives depend on the difference between returns on safe loans and returns on deposits. Table C1 illustrates the effects of greater risk-taking on the components of dividends for each realization of the aggregate returns. We map aggregate returns into states of nature and consider two cases depending on the sign of ε^*_{t+1}. The aggregate returns influence the value of the shield of limited liability. Risk amplifies the effect of the idiosyncratic shock. So, in every state of nature, the bank's choice of risk is determined by the expected effect of the idiosyncratic shock on the value of the shield of limited liability and returns on loans. The up-turn arrow, $\Uparrow$, indicates that greater risk-taking increases the corresponding component of bank's dividends. The down-turn arrow, $\Downarrow$, means that the corresponding component of bank's dividends decreases with greater risk-taking. Two arrows turned in the opposite directions, $\Uparrow\Downarrow$, signify that the effect of greater risk-taking is undetermined and depends the parameterization.

First, $\varepsilon^*_{t+1} > 0$ indicates that the bank makes losses on safe loans. It happens in those states of nature where the net income from the zero-risk portfolio is negative, so the bank is behind the shield of limited liability. By accepting more risk, the bank is more likely to get a positive net return under a favorable realization of the idiosyncratic shock as risk acts like a leverage on the size of the shock. Therefore, $1-G(\varepsilon^*_{t+1})$ rises. This balances with smaller returns on a portfolio with more risky loans, i.e. $R^s_{t+1} - R^d_t\left(1-\gamma_t\right) - \frac{\xi\sigma_t}{Q_t}$ goes down. Similarly, gambling on more dispersed returns allows the bank to move away from a zero return that comes from the limited liability to some positive return that is accompanied by less frequent defaults.

Table C1: Illustrating the Effects of Higher Risk on Dividends.

<table>
<tr><td rowspan="2">States of nature where</td><td colspan="2">Effects on ω_1</td><td rowspan="2">Effects on ω_2</td></tr>
<tr><td>$R^s_{t+1} - R^d_t\,(1-\gamma_t) - \frac{\xi\sigma_t}{Q_t}$</td><td>$1 - G(\varepsilon^*_{t+1})$</td></tr>
<tr><td>$R^s_{t+1} < R^d_t\,(1-\gamma_t) \quad \Leftrightarrow \quad \varepsilon^*_{t+1} > 0$</td><td>$\Downarrow$</td><td>$\Uparrow$</td><td>$\Uparrow$</td></tr>
<tr><td rowspan="2">$R^s_{t+1} > R^d_t\,(1-\gamma_t) \quad \Leftrightarrow \quad \varepsilon^*_{t+1} < 0$</td><td rowspan="2">$\Downarrow$</td><td rowspan="2">$\Downarrow$</td><td>if $\varepsilon^*_{t+1} > -\xi$, then $\Uparrow\Downarrow$</td></tr>
<tr><td>if $\varepsilon^*_{t+1} \leqslant -\xi$, then $\Uparrow$</td></tr>
</table>

So, the effect of σ_t on expected dividends from ω_2 is positive.

Second, $\varepsilon^*_{t+1} < 0$ shows that the bank makes positive profits on safe loans. The bank is more likely to default when it takes on more risk because any negative idiosyncratic shock would be amplified by risk. The bank internalizes that riskier projects are less profitable. Therefore, the overall effect of greater risk on ω_1 is negative when $\varepsilon^*_{t+1} < 0$.

Then consider the bonus from projects volatility. If $-\xi < \varepsilon^*_{t+1} < 0$, there are two contrasting forces. On the one hand, the bank always benefits from limited liability that makes the variance of projects returns attractive. On the other hand, the bank is more concerned about (and more vulnerable to) the variability of returns in the situation when taking on more risk would result in zero payoff instead of some positive payoff achieved by smaller risk. It occurs when $-\xi < \varepsilon^*_{t+1} < 0$. In these states of nature, the bank requires greater than average realization of the idiosyncratic shock in order to get a positive return. Call this type of shock a good idiosyncratic shock. This shock happens with probability smaller than 0.5. Define a bad idiosyncratic shock as a complement to a good idiosyncratic shock. An increase in risk increases the profits under a good shock. It captures the benefits from greater upside. At the same time, an increase in risk makes it more likely to get a bad shock. The bank trades off marginal profits coming from a good shock with marginal losses coming from the reduction of profits due to more defaults. Since the probability of the latter is greater than the probability of the former, the losses from defaults can dominate the benefits from greater volatility. This force goes in the opposite direction when $\varepsilon^*_{t+1} \leqslant -\xi$. The difference is that here the bank is more likely to get a good shock than a bad shock. Therefore, the bank puts more weight on the benefits from risk-taking than on its costs. It is verified mathematically that the effects of σ_t on ω_2 is unambiguously positive when $\varepsilon^*_{t+1} \leqslant -\xi$.

In sum, we find that net returns on safe loans, $R^s_{t+1} - R^d_t\,(1-\gamma_t)$, is the main driver for the bank's choice of risk. In the partial-equilibrium setting, we differentiate between three cases that characterize incentives for risk-taking.

First, $R^s_{t+1} < R^d_t\,(1-\gamma_t)$ applies to the states of nature where a relatively large negative aggregate shock is realized. Two forces against the one that seems to be of lesser relevance make the bank benefit most from taking risk. Second, $-\xi < R^d_t\,(1-\gamma_t) - R^s_{t+1} < 0$ applies to the states of nature where intermediate values (not too large and not too small) of either negative or positive aggregate shock are realized. There are more forces that lower incentives for risk. Third, $R^d_t\,(1-\gamma_t) - R^s_{t+1} < -\xi$ applies to the states of nature where a positive aggregate shock of a larger size is realized. Interestingly, there is a force associated with the bonus from projects volatility that makes it possible for the bank to increase risk. The choice of risk depends on the strength of that force, ω_2, relative to the negative exposure of returns from a loan portfolio to risk, ω_1. It still remains a quantitative question to find out how risk-taking is determined in the general equilibrium set-up.

Capital requirements affect risk-taking through a change in ε^*_{t+1}. When γ_t increases, ε^*_{t+1} falls. It means that the bank will be more likely to find itself in the states of nature where ε^*_{t+1} is negative. It forces the bank to keep more skin in the game, make the shield of limited liability less attractive and prevent the switch into financing risky projects.

D Consumption Equivalent Variation

Let $Welf^{opt}$ be the welfare level attained under the optimal policy:

$$Welf^{opt} = E\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}\right].$$

And let $Welf^{rule}$ be the welfare level attained under a simple rule:

$$Welf^{rule} = E\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(C_{\text{rule},t}-\kappa C_{\text{rule},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{rule},t}^{1-\varsigma_d}-1}{1-\varsigma_d}\right].$$

For given paths of consumption and deposits, we are interested in sizing a permanent tax Δ applied to the consumption utility stream under the optimal policy such that the level of welfare under the optimal policy with the tax is equal to the level of welfare under the suboptimal rule. Thus,

$$E\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(\left(1-\Delta\right)\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}\right]=Welf^{rule},$$

which can be rewritten as

$$E\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(1-\Delta\right)^{1-\varsigma_c}\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}\right]=Welf^{rule}.$$

Taking out $(1-\Delta)^{1-\sigma_c}$, we get

$$E\sum_{t=0}^{\infty}\beta^t\left[\left(1-\Delta\right)^{1-\varsigma_c}\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}+\frac{\left(1-\Delta\right)^{1-\varsigma_c}-1}{1-\varsigma_c}\right]=Welf^{rule}.$$

With some additional manipulations, we obtain

$$E\sum_{t=0}^{\infty}\beta^t\Bigg[\left(\left(1-\Delta\right)^{1-\varsigma_c}-1\right)\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+ \\ \varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}+\frac{\left(1-\Delta\right)^{1-\varsigma_c}-1}{1-\varsigma_c}\Bigg]=Welf^{rule}.$$

Let $Welf_C^{opt}$ be the welfare from the consumption utility stream attained under the optimal policy,

$$Welf_C^{opt} = E\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}\right].$$

Hence,

$$\left(\left(1-\Delta\right)^{1-\varsigma_c}-1\right)Welf_C^{opt}+Welf^{opt}+\frac{\left(1-\Delta\right)^{1-\varsigma_c}-1}{\left(1-\beta\right)\left(1-\varsigma_c\right)}=Welf^{rule}.$$

From the equation above, we can derive Δ, the consumption equivalent variation:

$$\left(\left(1-\Delta\right)^{1-\varsigma_c}-1\right)\left(Welf_C^{opt}+\frac{1}{\left(1-\beta\right)\left(1-\varsigma_c\right)}\right)+Welf^{opt}=Welf^{rule},$$

$$(1-\Delta)^{1-\varsigma_c} - 1 = \frac{Welf^{rule} - Welf^{opt}}{\left(Welf_C^{opt} + \frac{1}{(1-\beta)(1-\varsigma_c)}\right)},$$

$$(1-\Delta)^{1-\varsigma_c} = 1 - \frac{Welf^{opt} - Welf^{rule}}{\left(Welf_C^{opt} + \frac{1}{(1-\beta)(1-\varsigma_c)}\right)},$$

$$\Delta = 1 - \left(1 - \frac{Welf^{opt} - Welf^{rule}}{\left(Welf_C^{opt} + \frac{1}{(1-\beta)(1-\varsigma_c)}\right)}\right)^{\frac{1}{1-\varsigma_c}}.$$

References

Erceg, C. J., Henderson, D. W., & Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, *46*(2), 281–313.

Gertler, M., & Karadi, P. (2011). A model of unconventional monetary policy. *Journal of monetary Economics*, *58*(1), 17–34.

Guerrieri, L., & Iacoviello, M. (2015). Occbin: A toolkit for solving dynamic models with occasionally binding constraints easily. *Journal of Monetary Economics*, *70*, 22–38.

Van den Heuvel, S. J. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics*, *55*(2), 298–320.